A computer model of the tidal phenomena in Cook Inlet, Alaska
Robert F. Carlson
Charles E. Behlke

A COMPUTER MODEL OF THE TIDAL PHENOMENA IN COOK INLET, ALASKA

by

Robert F. Carlson, Ph.D.
and
Charles E. Behlke, Ph.D.

IWR-17

IWR 17

A COMPUTER MODEL OF THE TIDAL PHENOMENA IN COOK INLET, ALASKA

by

Robert F. Carlson, Ph.D.
Associate Professor of Hydrology
and
Charles E. Behlke, Ph.D.
Professor of Civil Engineering
Dean, College of Mathematics, Physical Science and Engineering
University of Alaska
Fairbanks, Alaska

INSTITUTE OF WATER RESOURCES
University of Alaska
Fairbanks, Alaska 99701

Report No: IWR-17

March, 1972

Index

Table of Figure Legends

ACKNOWLEDGMENTS

The work upon which this report is based was supported by funds (Project A-028-ALAS) provided by the United States Department of the Interior, Office of Water Resources Research, as authorized under the Water Resources Act of 1964, as amended.

CHAPTER 1 - The Modeling of Surface Water Bodies

Introduction

The present and impending industrial development in the Cook Inlet basin in southcentral Alaska (see map, Figure 1-1) is likely to cause numerous water resource engineering problems. If the usual pattern of development occurs, many of the problems will be related to industrial and municipal waste disposal. The problem potential within the whole basin is great, and in particular, the disposal of wastewater into Cook Inlet itself will receive increased attention. There is at the present time an increasing tempo of real and imagined conflict of interest between groups using the inlet as the final stage of waste treatment and those using it for recreation and other industrial uses.

If future engineering design within the Cook Inlet basin is to be directed towards minimizing water resource conflicts, a great amount of additional information will be needed. Of particular interest, we feel, is the transport and dispersion process of waste materials within the inlet itself. Due to the size of the inlet and its rather peculiar characteristics, the nature of this process is known only in a general way (Rosenberg et al., 1967).

The problem of defining the transport-dispersion process could be approached through field measurements, application of theoretical solutions, and application of an empirical modeling approach with some basis in theoretical concepts. The last is attractive from several points of view -- it can furnish solutions for cases not covered by field measurements and can indicate the most profitable location for future measurements. We have adopted an approach which concentrates on a simulation effort carried out on a digital computer.

The model described in this report is the first step -- a model of hydraulic flow which furnishes the basic data base for a dispersion-transport model. Future work will concentrate on Knik Arm in the vicinity of the city of Anchorage.

The field of hydraulic engineering which seeks to explain the dynamic behavior of natural bodies of water is quite well developed. In fact, in recent years, largely because of computers, a great variety of methods, techniques, and models relating to open surface water bodies have evolved. All of these attempt to explain or "model" in some fashion, the motion of water particles in a natural environment. As a result, it appears that in many cases the theory and technology of explaining the dynamics of large water bodies is somewhat ahead of the actual field measurements.

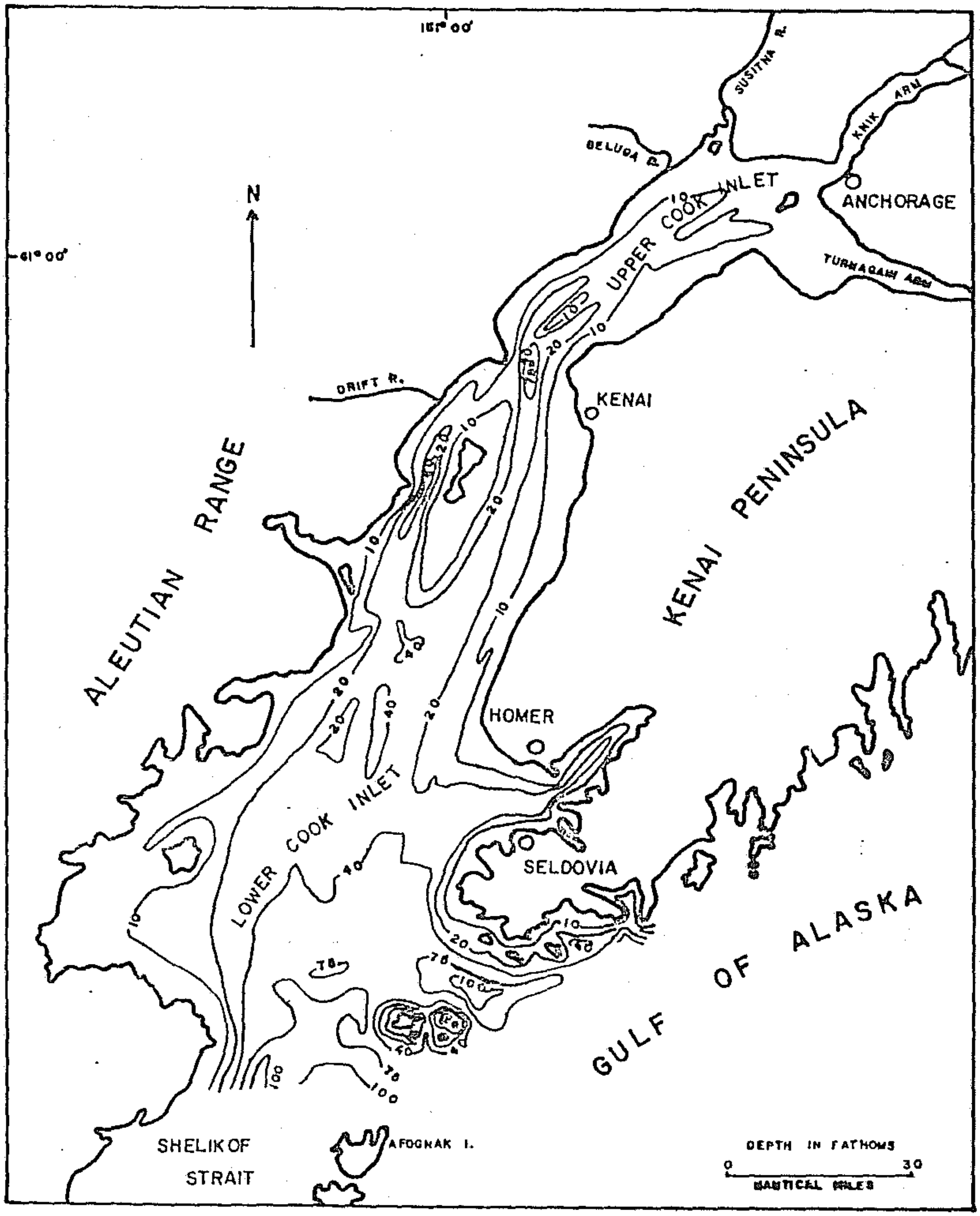


Figure 1 - 1. Location Map of Cook Inlet, Alaska.

When faced with a given problem, the engineer may choose an appropriate method, but he may also easily choose a method which is quite inappropriate and unduly burdensome with respect to computer time, expense, and complexity.

We have attempted to achieve a balance and have used a one-dimensional implicit method which will provide the necessary answers in sufficient detail and with a certain amount of flexibility, but will not be too burdensome in terms of cost and effort.

Scope of the Report

Cook Inlet is one of the most unusual bodies of water in the world. It is long, wide, and shallow, and is dominated by high tides which, in turn, cause high velocities. Physically, it is at a low temperature for much of the year and is continually mixed with a fine glacial silt. These and other factors make up an extremely harsh environment which presents many problems both to the existing natural marine life and to engineering development activities. It also presents many situations which are unlike any found elsewhere, and especially unlike the typical marine engineering activities along both coasts of the United States. For example, one might expect that the design of a sewer outfall might be based on quite a different set of criteria than that found, say, in an ocean current on the coast of California.

The objectives of this report, then, are to develop a tool which explains the important features of the hydraulics of Cook Inlet, form a background of flow information for a dispersion study, and look at several possibilities of changing the regime of the inlet, which are of interest themselves.

The characteristics of the flow in Cook Inlet and in Knik Arm are explained at the end of this chapter. Chapter 2 provides a brief introduction to analytic methods of describing tidal flows. Applications of the analytic method are made to Cook Inlet and are described in Chapter 2. Chapter 3, in combination with Appendix A, develops the numerical computer model upon which subsequent analysis is based. The application of the model is illustrated in Chapter 4. A summary of the report is given in Chapter 5. The remainder of this chapter is devoted to some background into the nature of flow modeling, which is useful to the understanding of the remainder of the report.

Some Background Information

The study of free surface hydraulics has been well established for a number of years. Early work in wave phenomena was done by Lord Rayligh in the mid-19th century and was developed extensively by Lamb beginning in the 1880's and continuing through the 1920's (see Lamb, 1945). The problem of finding solutions to unsteady flow problems is usually sought through solutions to the well-known wave equation (sometimes called "the shallow water wave equation" or the "Saint-Venant equations") for the appropriate set of boundary and initial conditions. The wave equation, in turn, is based on a set of two equations -- one which explains the continuity of mass of the flow, and the other which accounts for the conservation of momentum of the fluid. These equations, which are presented in more detail in Chapter 2, are usually written in terms of a flow variable.

This field of study is quite active and has developed a considerable amount of literature. An extensive review is not intended here, since the purpose of the study is to develop a tool for a specific purpose -- pollution transport in Cook Inlet. The reader, however, may want to obtain a more extensive background on certain aspects of the development. The list of scientific and engineering papers related to open-channel flow numbers in the hundreds. Books which develop and review the subject matter are: Stoker, 1957; Henderson, 1966; and Dronkers, 1964. Various chapters of Ippen, 1966, present a concise development and review of all apsects of estuary hydrodynamics. Particular use here is made of Chapter 10 by Ippen and Harleman. The most recent extensive review is Dronkers, 1969, which is essentially an updating of his earlier work. It is this paper upon which the flow model presented later in this chapter and Chapter 3 is based.

Each of the references listed includes an extensive bibliography for those who wish to study further.

Although the two equations of flow, Equations 2-1 and 2-2, appear to be rather simple and straightforward and are really quite simplified (see Dronkers, 1964, for a complete development of the equations), they are, in fact, insolvable as they stand. The problem has attracted researchers for the last one hundred years. Solutions have been sought from several points of view which can presently be grouped into two schools of thought.

One view has been developed by hydraulic or civil engineers whose traditional development has been with flows in land channels. The usual concern has been with highly transient flows with widely varying channel characteristics such as during a flood. Thus, the civil engineer tends to think of unsteady flow in open channels as a flow or discharge wave and would seek a solution to Equations 2-1 and 2-2 in terms of the velocity, u, or Q, the discharge.

Another point of view is that developed by ocean engineers and oceanographers, who concentrate on water motion and movements in the water surface elevation with a periodic-type motion. This view reflects their traditional concern with tide elevations and wave motions at sea. They, in turn, would seek solutions in terms of h, the water surface elevation.

The differences mentioned above are not really very important, except that one should keep in mind the wide variety of disguises and circumstances under which the same problem is discussed. If we completely define the whole elevation field along a channel, we must at the same time find the flow field.

The technique we adopt will depend on our interest and the measurements at hand.

The Nature of Flow in Cook Inlet

Cook Inlet is a large, wide, shallow estuary in Southcentral Alaska (approximately 61° N - 151° W) extending from Anchorage 150 miles southwest to the Gulf of Alaska (See Figure 1-1). The most famous characteristics of the inlet are its high tides (20- to 30-foot range) and strong tidal currents up to 4 knots (U. S. Coast and Geodetic Survey, 1967). These rather unusual conditions have attracted the attention of both engineers (Gaither and Dalton, 1969) and scientists (Zetter and Cummings, 1967).

The inlet is commonly considered as two parts - the lower inlet and the upper inlet, which are divided by a relatively narrow "forelands" 5 miles wide. The upper inlet is appended on the northeast end by Knik Arm and Turnagain Arm (see Figures 1-1 and 1-2). For a complete description of the physical and environmental characteristics of the area, the reader is referred to Wagner, et al., 1969. As mentioned above, we have a primary interest in Knik Arm because of the location of the main population center of Alaska, the greater Anchorage area, on its left bank.

The primary measurements of the tide and current regime in the inlet is presented by the U. S. Coast and Geodetic tide and current tables. The four primary measurements which characterize the simple sine wave type of tidal variation are the range (total distance between mean high and low water), the time angle or phase of the maximum tide height as measured from the time of high water at Cape Ninilchik, the maximum current velocity, and the time angle of the flow wave. The phase angles of the tide and flow waves are plotted as a function of the distance along the inlet, shown in Figure 1-3.

The hydraulic geometry of the Inlet cross section is shown for various stations in Appendix B. Special attention should be paid to a comparison of Figure B-3, which is a plot with the usual vertical exaggeration, with Figure B-6, which is a plot with the horizontal and vertical distances plotted to the same scale. The second plot indicates that the cross-section shown -- the deepest, narrowest section of the Inlet -- is indeed wide and shallow.

In addition to the measurements of the flow properties given by the Coast and Geodetic tables, Wagner et al. lists a number of other studies by federal, state, local and private agencies.

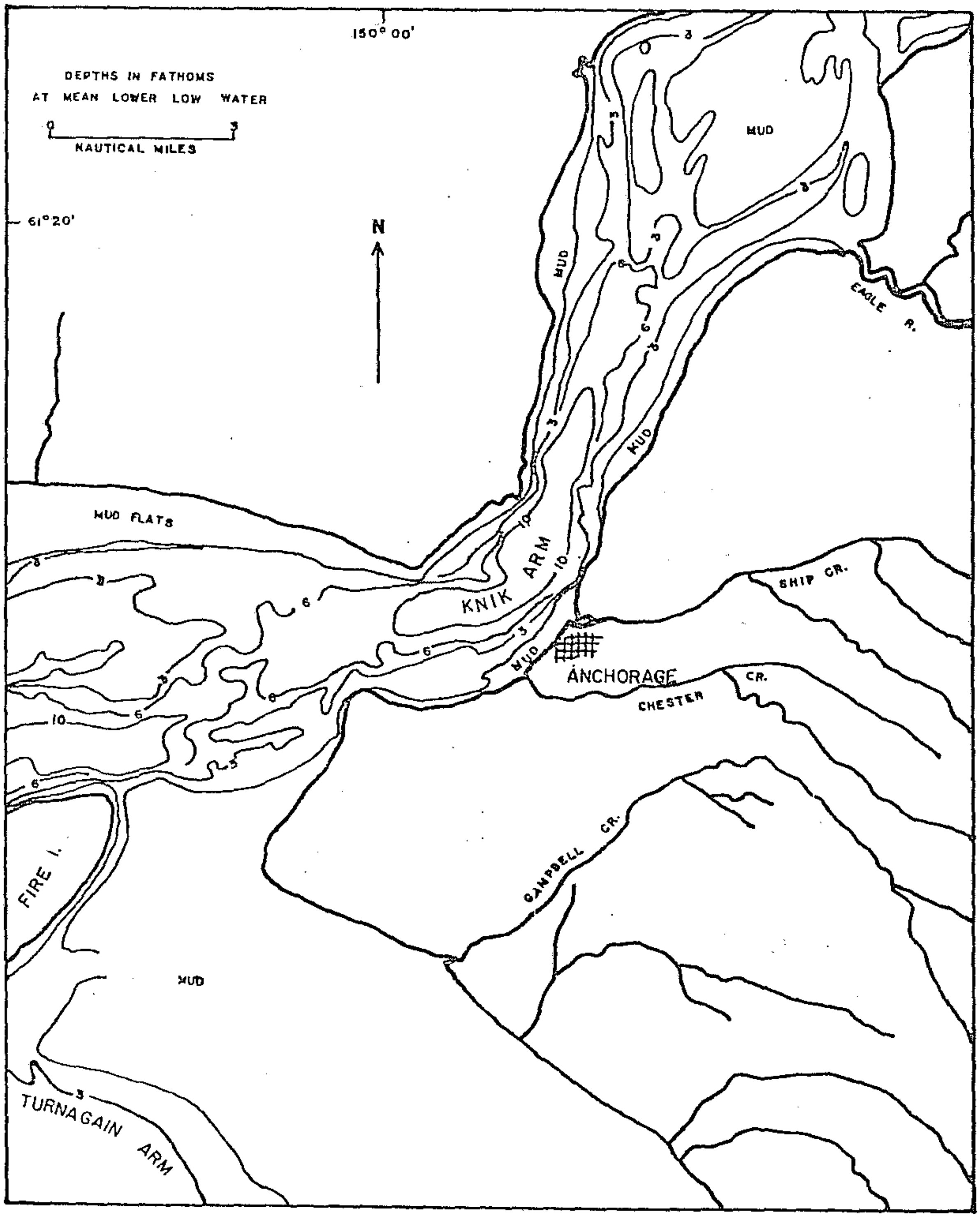


Figure 1 - 2. Location Map of Knik Arm

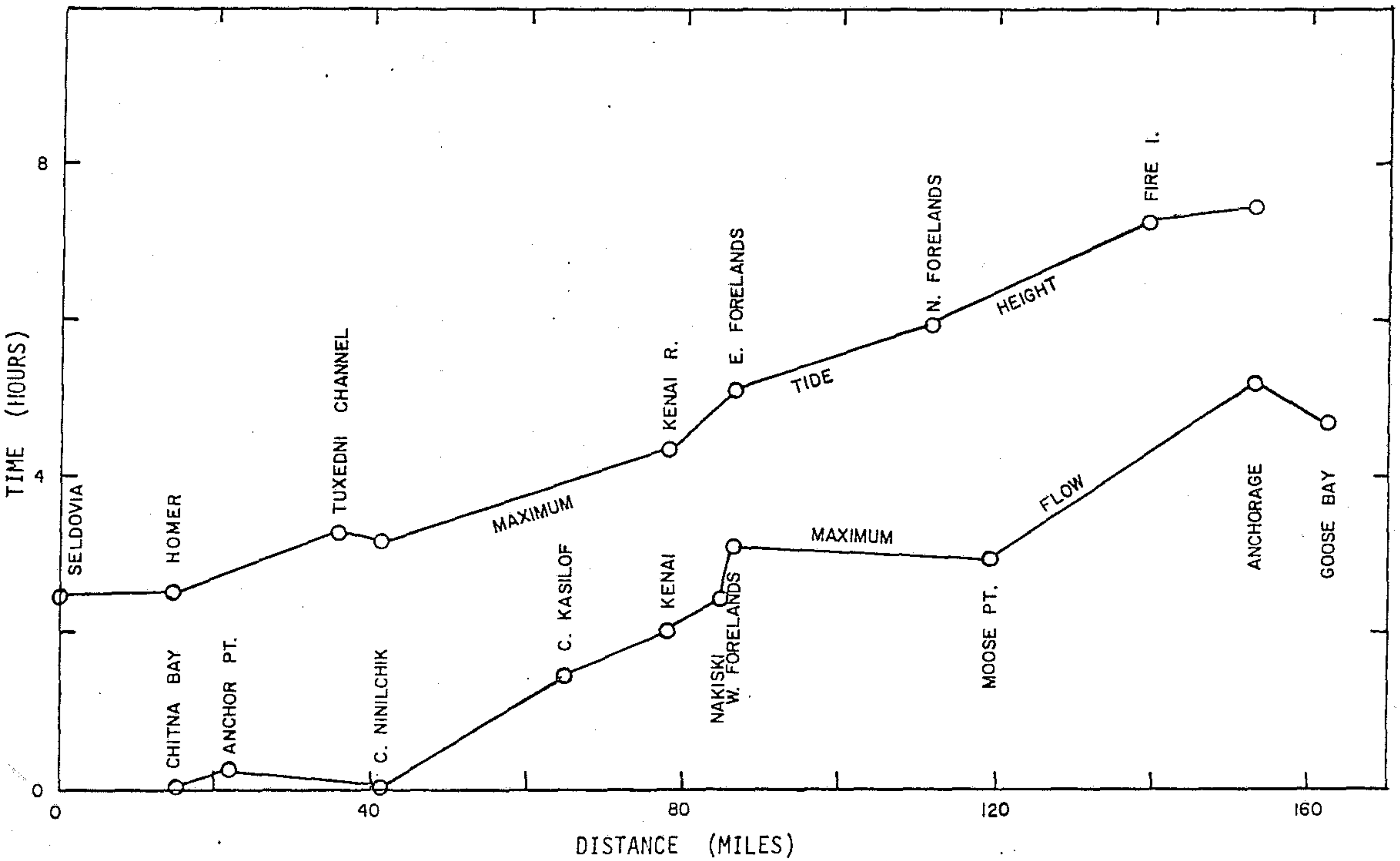


Figure 1 - 3. The time difference between time of high tide and time of maximum flow with reference to the time of maximum flow at Cape Ninilchik according to published field values for Cook Inlet.

CHAPTER 2 - The Analytic Model of Flow

Although this study is directed toward examining the flow in Cook Inlet through use of a numerical computer model, it is useful to spend a bit of time developing a few concepts of tidal hydrodynamics. A simple equation for expression of the various velocity and stage relationships is easily found for certain channel geometrics and the flow equations which are derived in this manner provide a basis for comparison for the numerical methods.

The description of water motion in an open channel must be based on two physical principles. The first, the continuity equation, requires that mass be conserved. The second, the momentum equation, a form of the familiar expression, F = ma, requires that the sum of forces on a body (F) must equal the rate of change of momentum (ma).

The equation of continuity is usually written as:

$$\underset{1}{\frac{\partial Q}{\partial x}} + \underset{2}{b\frac{\partial h}{\partial t}} = 0 \qquad \text{2-1}$$

which says that the rate of change in the discharge, Q, in the x-direction, term number 1, must be balanced by the rate of volume change caused by the rising of the water surface, term number 2. In this equation, Q and h, the water surface elevation, are the dependent variables of interest. The channel width, b, is a channel parameter and x and t are the independent variables, distance and time, respectively. (All symbols are identified in Appendix D with the appropriate dimensions.)

The momentum equation may be written in an abbreviated form as:

$$\underset{1}{\frac{\partial h}{\partial x}} = \underset{2}{-\frac{1}{gA}\frac{\partial Q}{\partial t}} - \underset{3}{\frac{|Q|Q}{C^2A^2a_o}} \qquad \text{2-2}$$

(See Dronker, 1964, for a complete development of the equation.) Equation 2-2 expresses the momentum principle which states that the sum of the external forces on a fluid element is equal to the rate of change of momentum of the element. Thus, term number 1 represents the pressure force of the fluid element (the driving force), term number 2 represents the momentum flux or rate of change of momentum, and term number 3 represents the friction drag (the restoring force) on the fluid volume. The additional parameters are: g, the gravity parameter; C, "Chezy's C", or roughness coefficient; A, the cross-sectional flow area; and a_o, the mean flow depth.

Equations 2-1 and 2-2 together form the basis for solution of the problem of finding Q and h as a function of time and distance along the open channel. They are usually called "the equations of motion for unsteady open channel flow."

The equations of motion may alternatively be written in terms of the velocity, u, and the water surface elevation, h, (Dronker, 1964), as:

$$\frac{\partial h}{\partial x} = -g \frac{\partial h}{\partial t} - g \frac{u \, | \, u \, |}{C^2 \, (a_\circ + h)} \qquad 2\text{-}3$$

$$\frac{\partial (uA)}{\partial x} = -b \frac{\partial h}{\partial t} \qquad 2\text{-}4$$

The analytic expression of the flow in an open channel, especially when viewed as a wave problem, has been long established. The classic in this field is Lamb's Hydrodynamics, 1945, the sixth edition of which appeared in 1932 and which first appears in 1879. Included in Lamb's volume is a 100-page chapter on tidal waves. Ippen, 1966, presents an up-to-date summary of wave theory at a level which is most adaptable to the needs of engineers. The various chapters of Ippen present numerous applications. The material presented below is essentially a condensed version of their chapter 10 by Eagleson. We do not intend to proceed with a complete development, but rather offer a few concepts of wave solutions which are useful as a basis for comparison for the numeric model solutions. First, a few terms and concepts are defined.

The tidal wave is considered as a long wave or as a shallow water wave. That is, the ratio of wave length, L, to the average depth, h, h/L_o is very small. The wave period, in this case the tidal period, T, is related to L_o via the wave speed of propagation, C. In the case of a long wave

$$C = (gh)^{1/2} = \frac{L}{T}$$

With this equation in mind, and considering a period, T, of 12.4 hours, = 44,700 seconds, we can compute C and L., for representative depths in Cook Inlet. For depths of 25, 50, 100, 180 feet, C equals 28, 40, 57, and 76 feet per second, and L equals 241, 338, 482, and 644 miles.

The usual formation of the problem is to take one of the sets of the equations of motion, Equations 2-1 and 2-2, or 2-3 and 2-4, and manipulate them to form either:

$$\frac{\partial^2 h}{\partial t^2} = C^2 \frac{\partial^2 h}{\partial x^2} \qquad 2\text{-}5$$

or

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2} \qquad 2\text{-}6$$

Either equation is in the form of the classic "Wave Equation" and has been used to describe the propagation of waves in a wide variety of media. The solution, of course, depends on the properties of the media, the boundary conditions and the excitation or forcing function. Sometimes the initial state of the system is alluded to, but this usually affects the system for only a short period of time. We will now look at two simple cases.

The Progressive Wave

If we assume that amplitudes are small compared to the depth (but still shallow water), water velocities are small compared to the wave speed, and there is no friction of the water with the channel boundaries, the solution for water surface elevation and velocity for periodic long waves entering a canal of uniform section and of infinite length is given by:

$$h = a \cos (\sigma t - Kx) \tag{2-6}$$

and

$$u = \frac{a}{a_o} C \cos (\sigma t - Kx) \tag{2-7}$$

where:

σ = the frequency number, $sec.^{-1}$
K = the wave number, $ft.^{-1}$
a_o = the mean depth, ft.
a = the wave amplitude, ft.

Note that u and h are in phase, of the same sign, and related by:

$$u = \frac{h}{a_o} C$$

This type of wave is called a progressive wave, and, as mentioned, makes up the simplest type of wave.

The Cooscillating Wave

Here we consider a channel which is closed at one end, causing a complete reflection of the wave energy at the closed end. The solution is given by an incident progressive wave:

$$h_1 = a \cos (\sigma t - Kx)$$

and a reflected wave:

$$h_2 = a \cos (\sigma t + Kx).$$

After some trigonometric manipulation, the resulting wave is

$$h = h_1 + h_2 = 2\, a \cos \sigma t \cos Kx, \text{ and} \qquad 2\text{-}8$$

$$\frac{\partial u}{\partial x} = - h \frac{\partial h}{\partial t} \quad \text{or} \quad u = \frac{2a}{h} C \sin \sigma t \sin Kx \qquad 2\text{-}9$$

Note that the velocity and tide amplitude are out of phase by 90 degrees. This is a characteristic of a perfect reflection. Also if length of the channel from the open sea to the closed end is given as $-l$, the ratio of maximum high water amplitude at the closed end, h_{max} to the maximum high water at the open end,

$$h_{(-l)}\max \text{ (at } x = -l)$$

$$\frac{h_o \max}{h_{(-l)}\max} = \frac{1}{\cos Kl}$$

Thus for $l/L = 1/4$, $Kl = \Pi/4$ and the above ratio becomes infinitely large. Cook Inlet has long been thought to be a "quarter wave length" long and therefore having high tides.

The application of these two simple cases will be illustrated below. It should be kept in mind, however, that we have discussed two very idealized cases which cannot be expected to yield good results when applied to an actual case. The departure from these idealized models is caused primarily by friction and changes in channel cross section. Ippen and Harleman summarize these effects as follows:

1. Friction will decrease the amplitude of the wave as it progresses up the channel.

2. A depth decrease or a width decrease will predictably increase the wave amplitude, but only if it is gradual enough to cause no reflection.

3. A more abrupt section change will change the amplitude and cause reflections which can be continuous or rather complex.

The procedure for using the above principles has been well established and can yield useful results. However, it was felt that Cook Inlet presented such a complex case of depth decrease, friction, and complex geometry that a direct application would be difficult. Therefore, at this point in the investigation, we decided to use a numerical model which uses the equations of motion, Equations 2-1 and 2-2, directly. It is still instructive, however, to look at a few simple cases without friction.

Applications of Analytic Model

As mentioned above, the analytic models without friction cannot be expected to simulate the flow in Cook Inlet very well. However, it is still instructive to look at a few simple cases. This both gives us some feeling for the mathematical equations themselves, and gives us some basis for understanding the flow we do have there. First we will apply the simple case of a progressive wave in that channel, and then we will apply the slightly more complicated case of a progressive wave in a channel without friction that will be closed in.

For the progressive wave we will use Equations 2-8 and 2-9, which gave the elevation of the water surface as a function of several channel and wave parameters. We will assume:

a) $H = 100$ feet

b) Range = 18 feet; therefore $a = 9$ feet.

Using these values, we can compute the following:

1. $C = (gh)^{1/2} = 57$ feet/second

2. $\frac{a}{h} = \frac{10}{100} = .1$

Therefore:

$$u_{max} = C \frac{a}{h} = 5.7 \text{ feet/second}$$
$$= 3.4 \text{ knots}$$

This compares with data from the tide tables which gives a maximum velocity of 2.4 knots at Anchor Point and 3.0 knots at Cape Kasilof. As the two equations indicate, u and h are in phase, that is, the time of maximum velocity and high water or of maximum elevation occur at the same time, or, assuming that a tidal wave corresponds to a sign curve at time of slack water, then high tide would be ninety degrees out of phase or 3.1 hours. A look at an example of the time differences between high and low tide and maximum flood and ebb occurence indicates the following values: for March 5, 1968, the average difference between high tide and maximum flood was three hours and twelve minutes, the maximum average difference between the low tide and maximum ebb was four hours and two minutes, or an average difference between high and low tide and flood and ebb of three hours and thirty-seven minutes. Likewise, the average difference between slack water and high and low tides was approximately twenty-seven minutes one way and twenty-three minutes the other, or approximate average of zero minutes, which indicates that slack water is approximately in phase and occurs at the same time as either high or low tide.

These values indicate that although a progressive wave model predicts the currents rather well in Lower Cook Inlet, it does not predict the time phasing of the current versus the tide and the water surface elevation. In fact, as we see below, the ninety-degree phase angle indicates a rather good reflection, perhaps at the Forelands.

A Co Oscillating Tide and Application to Lower Cook Inlet

Since the application of a simple progressive wave did not seem to entirely fit the data which is available in the Lower Cook Inlet, and this in fact suggests a rather strong reflection off the Forelands area, we will now apply the co oscillating tidal model directly to Lower Cook Inlet and treat the Forelands area as being the closed end, and as $x = 0$, with the distance down the Inlet from the Forelands being treated as a negative distance. We will use the two equations, 2-8 and 2-9, which indicate respectively the wave height and velocity at any point, x, in the channel as a function of time, t. It should be noted in the derivation of equation 2-8 two progressive waves of amplitude, a, were added, creating a combined wave which has a maximum amplitude of 2a, or a range of 4a. The distance from Seldovia to the Forelands, 88.4 miles, is taken as l, with L or Ct computed as being 483 miles and a quarter wave length, L/4, is therefore 121 miles. It can be seen, then, that the l which we have in the present case is quite close to the quarter wave length, and we would expect then rather high reflection values in the vicinity of the Forelands. A computation of the previous section indicates that the maximum amplitude at $x = 0$, will be 2a, and the maximum amplitude at $x = -l$ will be $h_{-l} = 2a \cos Kl$ and

therefore a ratio of maximum amplitude at the Forelands to maximum amplitude at Seldovia is given by:

$$\frac{h_o}{h_{-l}} = \frac{1}{\cos Kl}$$

and if

$$l = 88.4 \text{ miles}$$

$$K = \frac{2\Pi}{L} = \frac{2\Pi}{483}$$

$$\cos Kl = \cos 66° = .405$$

Therefore

$$\frac{h_o}{h_{-l}} = \frac{1}{.405} = 2.47$$

By comparison, the actual tidal range at the tidal amplification factor between Seldovia and the Forelands is equal to:

$$\frac{\text{Range at Forelands}}{\text{Range at Seldovia}} = \frac{18.0}{15.4} = 1.17$$

Thus the amplification predicted by the co-oscillating model is over two times that which actually exists. Another way of looking at this would be that, according to the model, the tide range at the Forelands would be 15.4 x 2.47, or approximately 38 feet. As indicated previously the phase angle,between the maximum velocity and maximum high water is rather close to that which actually exists, that is, approximately ninety degrees. For a further check of the accuracy of the model, we will compute the maximum velocity at Seldovia. Here we use equation 2-12 to give the following result:

$$u_{max}(-l) = \frac{2a}{h} \sin 66^\circ$$

$$= \frac{2 \times 4.5(.9)}{100} = .1 \text{ feet/second}$$

This value is much lower that the field value of 2 knots given in the Inlet at the Seldovia area.

Conclusions Concerning Adequacy of the Progressive and Co Oscillating Models

The above results indicate that a progressive wave represents the tidal phenomenon in Lower Cook Inlet rather well, that is, a wave travelling up the Inlet with little reflection. It has two discrepancies. One is that there is a slight amplification from Seldovia to the Forelands which the progressive model does not predict. The other is that the current and elevation waves are approximately ninety degrees out of phase, which is contrary to what the progressive model predicts. The progressive model does predict the velocities rather well.

The co oscillating model, although it coincides almost exactly with the prediction of ninety-degree phase angles between current and elevation waves, it does rather poorly in predicting the tidal amplification and the velocity. Two other factors in both models which we should keep in mind is that we do not include the effects of friction or the effects of changing depth. Examination of the cross-sectional data indicates rather clearly that there is certainly a decrease in average depth as one proceeds up the Inlet and that we might expect friction to be present.

CHAPTER 3 - Numerical Computation Model

The previous chapter discussed a set of analytic models which may be applied toward understanding the nature of hydraulic phenomena of Cook Inlet. As we saw in that chapter, the models were quite suitable in some respects, however were quite deficient in others. We restricted ourselves to looking at simple frictionless models, and saw that they explained many aspects of the tidal phenomena, and gave us a base against which to compose future studies.

These analytic models could have been extended to include friction. This method is outlined in some detail in Ippen, and therefore will not be explained further. The advantages of the analytic model, especially one that includes friction, would be that the result would be in the form of an equation, which is an advantage when studying various types of input and flow relationships. Also, it is rather straightforward to express the time-phase relationships among the various combinations of flow and elevation waves. However, rather than continue with this line of attack, we selected instead to develop a numerical computation scheme.

In spite of the many good reasons for using the analytic model, we felt that the numerical model favored the present situation for several reasons. First, Cook Inlet represents a rather complex geographic configuration, being divided into two rather definite basins with a fairly narrow connection in between. Second, the existence of the two rather shallow arms, Turnagain and Knik, at the upper end of the upper part of Cook Inlet proper, represents a configuration which is very difficult to handle in the analytic model, especially if one wishes to study the nature of flow in this region. Third, because of the shallow water depth and rather high tides, the use of the linear friction term would appear to be rather unrealistic. Fourth, the depth and side convergence is greater than that usually allowed by Green's Law, upon which the Ippen-type procedure is based. Fifth, although the data in Cook Lnlet is fairly detailed, it is quite lacking in some regions, especially near the upper half. As explained later on, we intended to look at Knik Arm itself and here we only had one tide station, and therefore not really enough data for the detailed fitting of the analytic model. In summary then, we felt that the analytic model was not to be favored over a numerical computation scheme, in spite of its rather good advantages, because of the involved nature of the fitting procedure. It may, in fact, hide several features of flow in which we may be interested. Perhaps the main advantage of the numerical model over an analytic scheme is that it offers a great deal of flexibility both for looking at a variety of input conditions, such as differing rates of freshwater throughflow, extreme tides at the ocean end, and the results of constriction to the flow, such as a causeway across one of the arms.

Numerical Modeling of Open Channel Flow

A great deal of material has been written on the various methods of numerical modeling of open channel flow phenomena. A particular example which is especially useful in understanding the type of problem at hand has been summarized in Ippen, Ligget and Wolheizer, and Dronkers.

It is pointed out in these references that a numerical computation scheme which is intended to model open channel flow may usually be classified as being a variation of one of three methods -- a characteristic method, an explicit method, or an implicit method. A characteristic method depends on a generalization and computation from the so-called "characteristic" equations. The "characteristic" equations themselves can be used in a variety of methods, but in this context it refers to a numerical solution. Explicit methods refers to methods which write the difference equivalent of the differential equations 2-1 and 2-2 in a rather straightforward way.

Implicit methods also write the difference equivalent of the differential equations. However, in the implicit methods, the equations are written and the solution carried out so that certain advantages are incurred in regard to stability of the solution. The disadvantage of implicit methods are that they usually require a somewhat larger storage of computer capacity to carry out.

The advantages and disadvantages of each method are not exactly clear-cut. For example, Ippen indicates successful results for each of them, and Dronkers has reported on a complete evaluation of all three methods and outlines certain advantages and disadvantages for each.

The advantages and disadvantages in Dronkers is dependent upon the case at hand. Ligget and Wolheizer (1966) make the rather strong recommendation that the "characteristic" method is to be preferred for computations of open channel flow, with perhaps the "implicit method" preferable in some cases. They indicate rather emphatically that "explicit" methods are to be avoided as they are unstable and unreliable in almost all cases.

The numerical method chosen for this study is that described by Dronkers (1969) in which he terms his third "implicit" method. Hereafter it will be called "Dronkers' third implicit method." The details of the complete derivation of the algorithm are left for Appendix A, and for reference to Dronkers (1969). The succeeding sections will describe in intuitive fashion some of the highlights of the method.

Dronkers' Third Implicit Method

The solution again begins with the equations of motion, equations 2-1 and 2-2. These equations of motion are written in terms of differences of quantities with respect to time and space, rather than as differentials.

This step quite seemingly is rather straightforward, but should not be taken lightly, as it immediately establishes a new set of equations. Although the difference equations should closely represent the differential equation, the procedure from the continuous variation of time and space to a finite variation is a fairly large jump and must be done with a great deal of care. The solution of the difference equations is often times not too difficult, and can always be found. The problem is that we may be finding the correct solution to the wrong problem. The point of all this is that the step from a differential to a difference equation is quite important, and should be carried out with the utmost care. This is probably true in any numerical modeling of physical phenomena, but is especially true in open channel flow which forms perhaps the most difficult physical situations from the standpoint of attempts of computer modeling. As stated above, there are a number of ways in which this can be carried out and within the three broad categories stated above, a wide variety of procedures. We found that Dronkers' third implicit method appeared to be as good as any other, and somewhat more flexible. It seemed to fit the condition we have had at hand.

The first step in implementing the different solutions is to split the channel into several segments. The more segments, the better, from the standpoint of a realistic modeling of the physical phenomena; however, the more segments, the longer the computation, and the more complex the model. Once the channel is split into the several segments, the next step is to write the difference equivalents of the pair of differential equations for each segment. Thus, if on a given channel, ten cross-sections were specified, nine segments are formed, and nine pairs of equations can be written. This in conjunction with the pair of variables of each of the segments forms a set of eighteen equations and twenty variables. Two of the variables are specified as functions of time, which we call "forcing functions", and the result is an eighteen variable, eighteen equation, solution matrix.

The solution procedure progresses in a series of time steps. At the beginning of the first step of time, we must of course specify a set of initial conditions. The solution for that step in time, then, is reached when we have calculated the conditions which will result at the end of the time step. Dronkers' third implicit method proceeds via a series of sweeps. First, the relationship is established between the variables such that a series of coefficients can be calculated. Then, beginning with the variables at one end of the reach, coefficients are calculated for each segment in turn, as one proceeds down the reach. At the other end, another boundary or set of boundary conditions is given, or the variables computed at the last cross-section. The unknown variable of the pair is computed, and then the computation proceeds back down the reach on the second sweep. When the starting point is then reached, a set of eighteen new variables are known -- that is the calculated values of the eighteen variables are known and recorded. The computation then proceeds to the next time step. The calculated variables from the previous time step become the initial conditions for the second time step. The computation then proceeds exactly as before with two sweeps. The computation proceeds with a series of time steps in this manner.

It is appropriate to point out a particularly intriguing and troublesome feature of the solution. We usually have a need to compute or model the flow via the numerical method for a certain length of time, keeping in mind that we are only approximating the differential equations or solutions to the differential equations. We would like to, for the best approximation, choose the smallest element of time feasible, with the knowledge that if we choose a larger element of time, the approximation is likely to be less good. However, if we choose as small a length of time as we would like, it may take an inordinate amount of time to carry through the computation of the period of time desired. Therefore, we have to choose from lengths of time such that the approximation to the real problem is close enough, but yet not so unnecessarily close that excessive computation times are used. The one great advantage of the "implicit" method becomes apparent at this point. These methods are inherently stable. That is, no matter what time step we choose, the computation of new variables will proceed in such a way that if an error is made in one direction, it will not continue to propagate through the solution. This is not to say that a solution of large steps is as accurate as one at smaller time steps, but that we do not have to worry about undetected instabilities within the computational program. This is not too serious when we are talking about a ten to twenty segment, one-dimensional model, such as we apply in the next chapter. But, if we were to extend this to a two-dimensional one, and therefore must keep track of several hundred variable points, it would nearly be impossible to ascertain whether or not one does not have the superfluous errors wandering through the computation scheme.

CHAPTER 4 - The Investigation Program

General Nature of the Investigation Program

Previous chapters described in detail the nature of the hydraulic regime of Cook Inlet, Alaska. The present section will be devoted towards describing the investigation program itself.

The investigation program, in this case a series of computer runs, was carried out for two purposes -- 1) to test the validity of the computation program. A great deal of effort was not devoted towards experimenting with various numeric computation schemes as it is felt that the computation algorithm used, the Dronkers' Third Implicit Method, was quite suitable for the work at hand and did not need further testing. Nevertheless, there is a possibility for a great number of the errors in any computation program and it becomes necessary to test the program. Also there is a need to adjust certain parameters in the model to the specific configuration being modeled, and 2) because, as mentioned previously that, our primary purpose is to model the water quality, a certain amount of flow information must be generated.

The investigation program can be split into three phases: 1) an initial verification of the model, 2) study of the whole of Cook Inlet, and 3) a study of Knik Arm in the vicinity of Anchorage.

The first may be described very briefly. Data as given by Dronkers was programmed for the model and results checked. Several computational errors were discovered and corrected. After a few data runs, the model performed satisfactorily and verified data, as reported by Dronkers. The program for phases two and three are reported on at some length in the remainder of this chapter.

In an attempt to look at an application with a fair amount of data, the model was first applied to Cook Inlet proper, from Seldovia to Fire Island. Use of this reach required selection of certain boundary conditions. For most of the runs, we treated the upper end in the vicinity of Fire Island as being closed, and the end at Seldovia as varying, as a sine wave. These are both rather restricted conditions and certainly do not represent the true nature of hydraulic flow. Nevertheless, it does enable us to devote our efforts towards certain features of the model and not have to worry about these boundary conditions. These assumptions are not too restrictive, as the volume of water in Turnagain and Knik Arms are rather fairly insignificant when compared to the Inlet as a whole. The tidal assumption at Seldovia is the best one can make without adding a great amount of complication to the model. Note at this point, that of the four boundary conditions that exist at the ends of the model, we must specify two -- the tide height variation at the lower end, and flow variation at the upper end. With only small modifications of the computer program, one could specify any combination of the four tide heights and discharge at either end.

To measure the success of the modeling effort, we have used two indicators -- the variation of the mean tide range as a function of axial distance along the Inlet, and the estimated mean discharge as a function of distance. The tide height range tends to be quite reliable as we have a rather good record along the Inlet. Also the conditions at the shore tend to be quite representative of the variation further out. The other indicator, the maximum flow value, has not served the purpose quite as well. The measured values are usually rather close to shore and offer a rather distorted indication of the flow further out. Therefore we have fitted the mean tide range to the parameters of the model and have used the natural flow value as an additional indication. These two parameters are indicated on Figures 4-1 and 4-2, and are labelled as the actual values. A number of computer runs were made for this section of the Inlet. A representative result of the model are given by run number 11 and run number 13. The results are shown in Figures 4-1 and 4-2.

The Investigation Program

The parameters specified for each run are indicated in Table 4-1. As indicated in Figure 4-1, the model duplicated very well the mean tide variation as a function of distance for the entire Inlet. There is some departure from the actual values in the vicinity of the Forelands. However, considering the assumptions made, these are not serious.

This particular example serves as a good illustration of the versatility of numerical models. Because we were able to completely specify the geometrical nature of the Inlet and reasonable boundary condition values, the only parameter we had to "fit" was the roughness coefficient for the channel bottom. We first choose a value of 100 for Chezy's C, and then a value of 120.

The use of the other indicator, variation of maximum flow as a function of distance, did not serve as well the purposes of the experiment. The results for runs number 11 and 13 are shown on Figure 4-2. The fit is not all that bad, but, nevertheless, does not provide an extremely good relationship to the values indicated. Although this may indicate inadequacy of the model, or maybe an inadequacy of our boundary conditions, we feel that it can be contributed to the fact that the point variations of velocity at the shore does not represent the average velocity across the inlet.

Another example of the model results may be seen by examining the variation in time angles between the maximum high tide and of maximum flow at various points along the Inlet. The results of runs number 11 and 13 are indicated in Figure 4-3. Here again, the maximum high tide fits quite well up to the Forelands Region. We attribute the departure past this point to our boundary condition at Fire Island. That is, we closed off the Inlet at this point and caused a maximum reflection when in fact only a partial one existed.

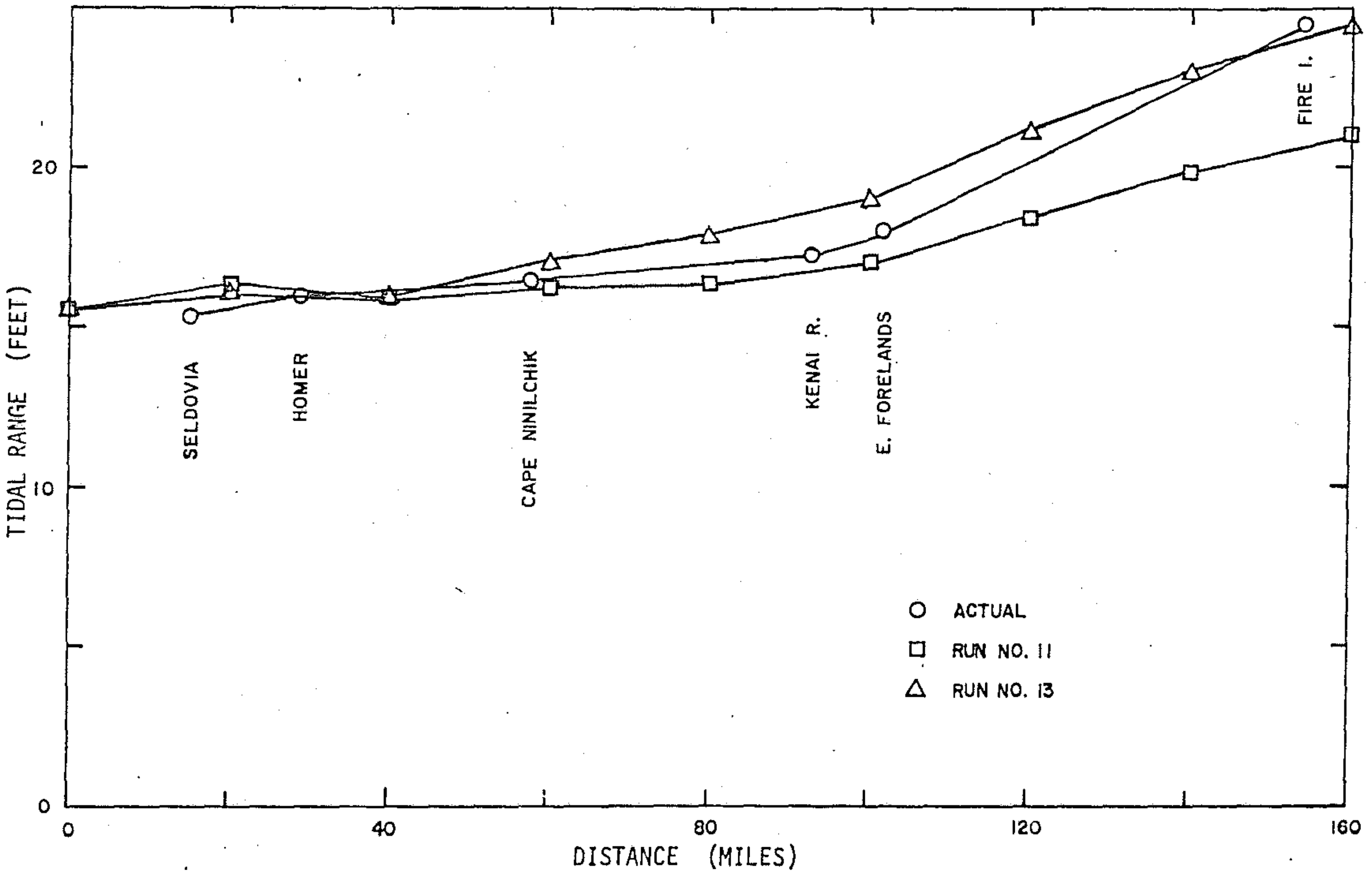


Figure 4 - 1. The mean tide range as a function of distance for Cook Inlet -- field and computed values.

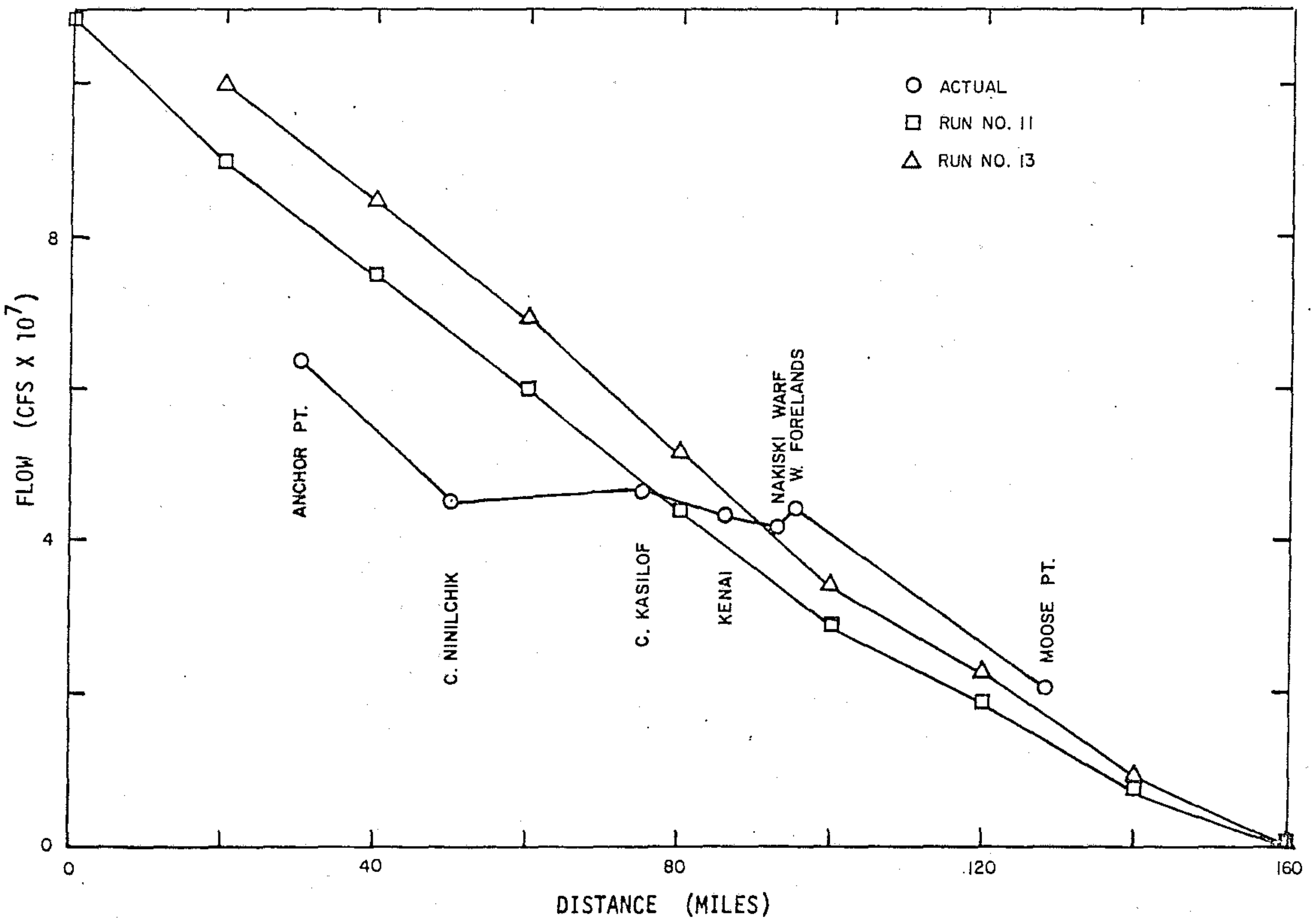


Figure 4 - 2. The maximum flow value as a function of distance for Cook Inlet -- field and computed values.

Table 4 - 1

Summary of Investigation Parameters

Run Number	Location	Q at end	Chezy's C	Dist. Step
11	Cook Inlet	0	100	-
13	Cook Inlet	0	120	-
17	Knik Arm	0	120	5,000 ft.
22,24	Knik Arm	-11,256	120	10,000 ft.
23	Knik Arm	0	120	10,000 ft.
28	Knik Arm	Sin Wave Flow Q_{max} = 1.2 million cfs.	120	10,000 ft.

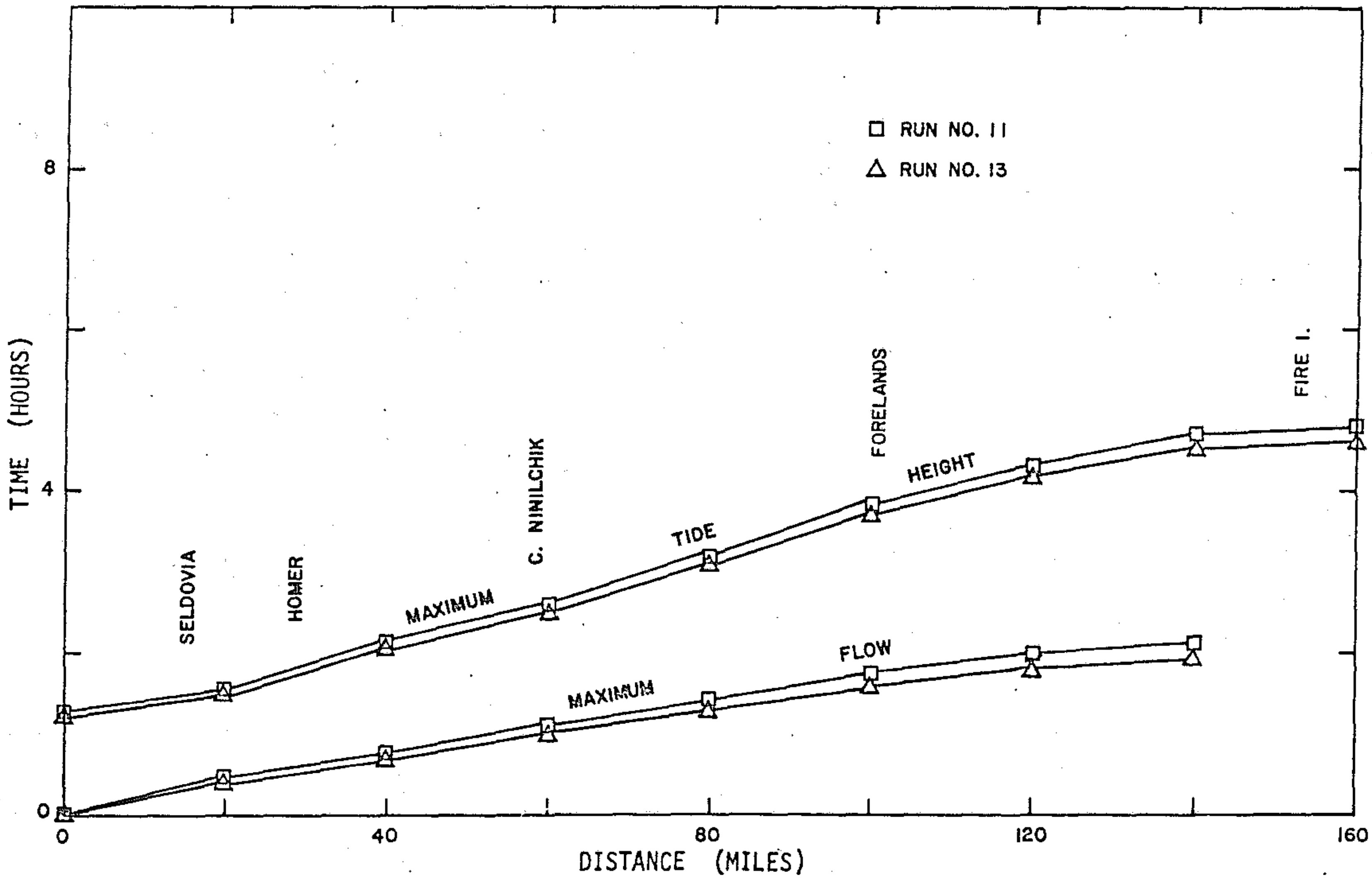


Figure 4 - 3. The relative time difference between time of high tide and time of maximum flow given by the computed values of Runs 11 and 13.

In summary, the results of the initial computational effort as applied to Cook Inlet may be stated as follows: the preliminary results indicate a rather good fit to published data. The departures of the model data (due to inadequacies), can for the most part be explained by the non-representative nature of the available data. With more effort in this regard, perhaps a satisfactory fit could have been made. The model experienced especially good results when attempting to match the variation of tide height, both time angle and range, along the channel, but it experienced rather mediocre results when attempting to fit the maximum flow. Although this could indicate an inadequacy in our model or perhaps unrepresentative boundary conditions, we prefer to believe that the results indicated in the published flow data for the vicinity of the shores is not representative of the Inlet as a whole.

The overall results suggest that the computer model is adequate for the task at hand, that is, it gives a fair representation of the gross one-dimensional flow of the Inlet, and that it is capable of handling a rather wide variety of geometric configurations and boundary conditions, although we did not choose to explore this advantage in this case.

Final results of the actual computer program are plotted out for one tidal cycle at Seldovia for Runs Number 11 and 13, in Figures 4-4 and 4-5.

Application to Knik Arm

The primary purposes of the model study are to determine the hydraulic regime of Knik Arm, which is located at its upper end (Cook Inlet), off-shore from the Anchorage metropolitan area. For our purposes, we will consider Knik Arm as beginning at Fire Island.

Knik Arm is important to the total Cook Inlet environment for several reasons. First, it is the main region of interest to us, from the standpoint of developing a water quality model, as it is into this waterbody that the metropolitan Anchorage area discharges its sewage. In addition, much of the freight shipment in Alaska comes in through Knik Arm, and it is the site of many proposed projects for pipeline and causeway crossings. We feel that much of the additional marine construction activity in Alaska will take place in or near its vicinity.

Knik Arm presents a different set of problems with but a few similarities to that of Cook Inlet proper. Practically no data exists on the variations of the distribution of tide range and discharge amplitude with distance along the arm. Tide stations exist at Fire Island and in Anchorage and current measurements have been made off the dock in Anchorage and at Goose Bay. Also a rather complex boundary problem exists in Knik Arm immediately upstream from

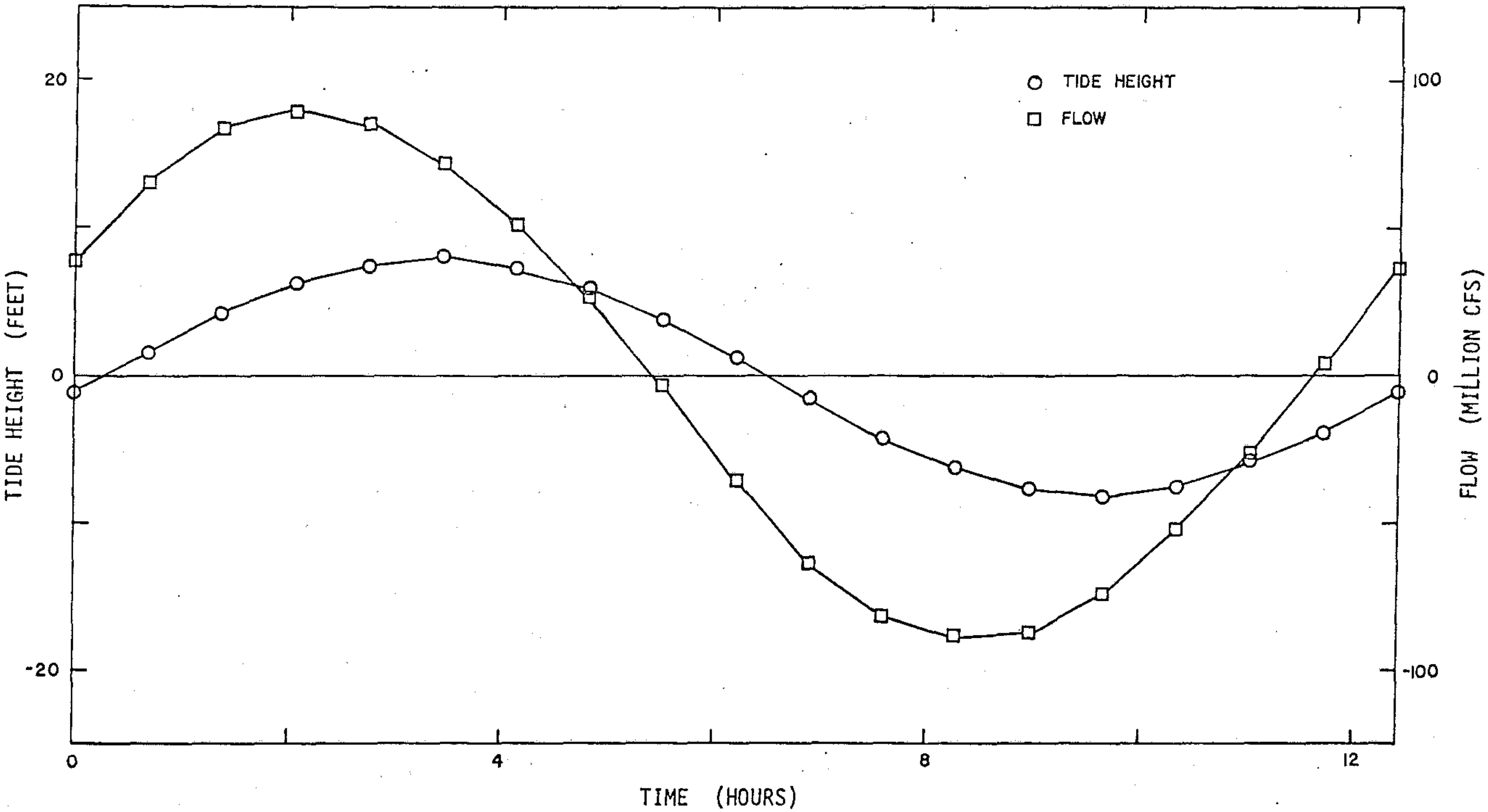


Figure 4 - 4. The computed tide height and discharge as a function of time for one tidal cycle at Seldovia, Run 11.

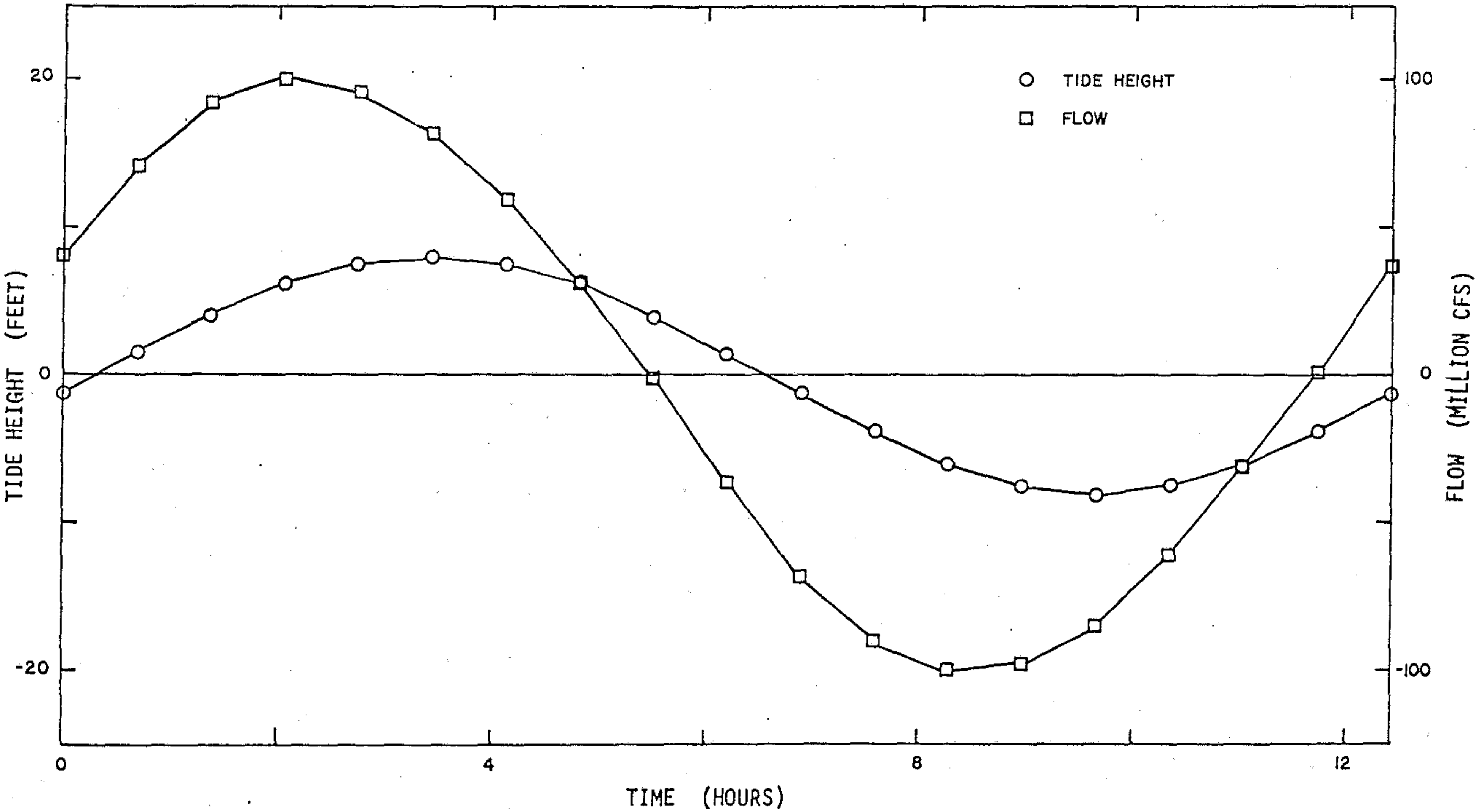


Figure 4 - 5. The computed tide height and discharge as a function of time for one tidal cycle at Seldovia, Run 13

the Anchorage area where it narrows to half its width. Immediately upstream from these narrows, shoaling begins which leaves a great expanse of mud flats exposed at low tide. Thus, we have a rather complex boundary condition which is difficult to treat with even a more complicated model. Nevertheless we feel that the simple one-dimensional model is capable of revealing a good bit of information on the hydraulic regime of the arm.

The investigation program consisted mainly of experimenting with a number of boundary conditions. We did not vary Chezy's C, but held it at a value of 120, as indicated by previous experiments of Cook Inlet proper. Boundary conditions exist as two types -- those of the actual flow boundaries, which are established by the specific cross-sections, and those which are specified by pre-supposing conditions of the variables at the two end reaches. Here again we must specify two of the four variables at the end reaches. We chose to specify the tide height variation at Fire Island. This variation with time was again considered to be variable, as a sine wave with a mean range as given from the published data. At the other end, various schemes were used to specify the tide height. One, the discharge was set equal to zero. That is, this is considered to be a perfect reflection at the boundary. Second, the mean flow was specified, at 11,256 feet^3 per second. Finally, a flow wave was put in at the upper boundary which was 90° out of phase with the tide wave at Fire Island, a perfect reflection characteristic. It had a maximum amplitude of 1.2 million cubic feet per second. In addition to specified various flow conditions at the upper end, we experimented with a number of different types of flow boundaries. None of these proved to be entirely satisfactory. The final results are shown in Figures 4-6 and 4-7. As the information in Figure 4-6 indicates, the tide range as a function of distance did not vary a great deal over a wide variety of runs. On the other hand, we had a great variation of the maximum flow value as a function of distance as indicated in Figure 4-7. The reason for the tide range being rather insensitive is because of the great dominance of the specified wave at Fire Island. Thus, although we do not place a great deal of confidence in the measured flow values at Anchorage and Goose Bay, we chose to adjust the model so that we obtained a fit at those points. As indicated in Figure 4-7, the most satisfactory results were given by Run No. 28 and Run No. 31.

An example of the actual computer results is in Figures 4-8, 4-9, 4-10, and 4-11.

A complete evaluation of the adequacy of the model must await more adequate data. However, with the limited data available, we felt the results indicated that the model was essentially correct and tended to be sufficiently flexible to allow a fitting of a variety of boundary conditions. For example, we were able to specify the tide variations at Fire Island, and could have chosen any number of boundary conditions at the other end. If we had known the tide variations, we could have specified these, and then left the flow values to be computed. The actual path that we chose in applying the model may not be the most appropriate in every case, but we feel that it does serve to illustrate the model as a tool for understanding the hydraulic regime of a rather complex inlet.

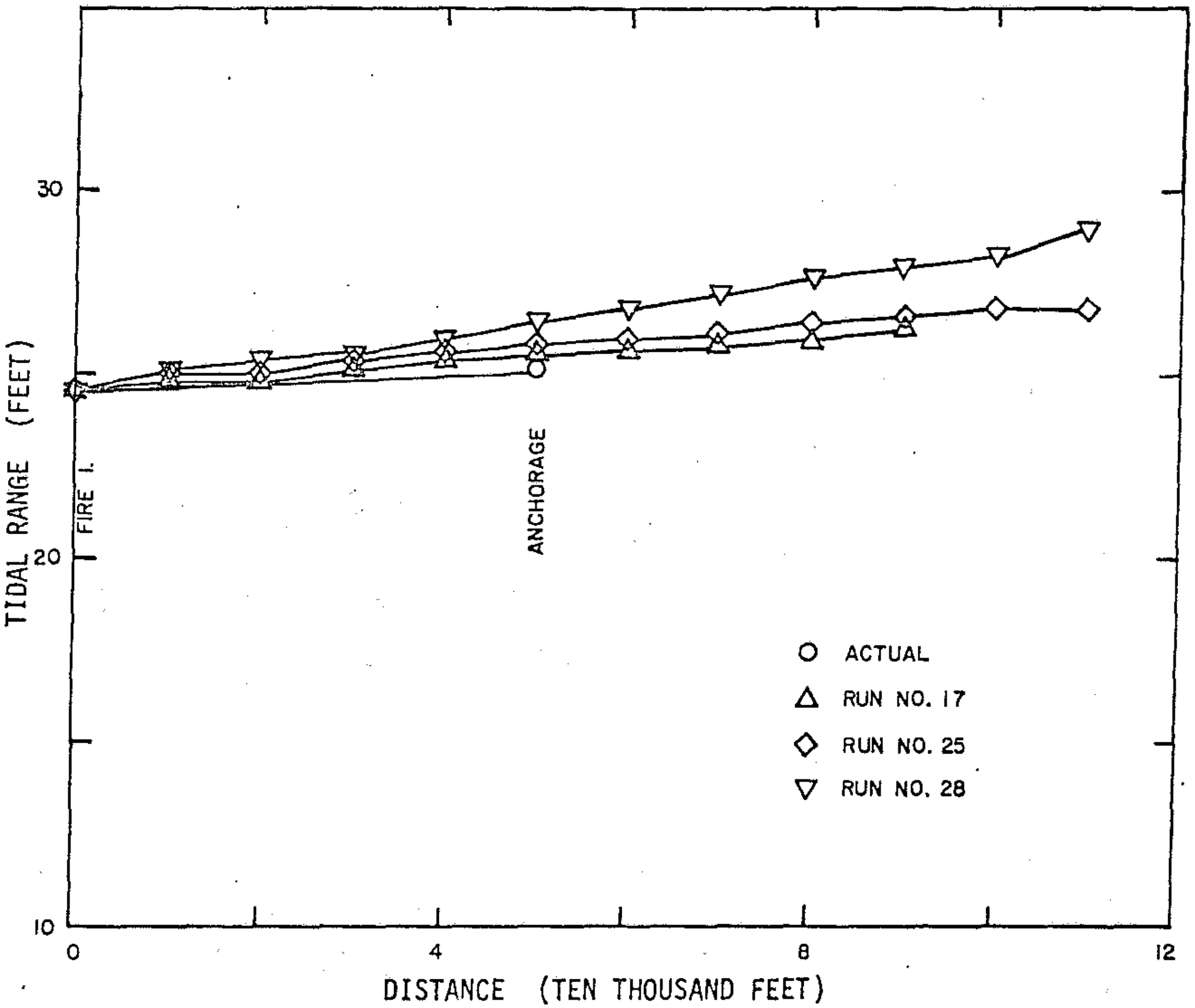


Figure 4 - 6. The mean tide range as a function of distance for Knik Arm -- field and computed values.

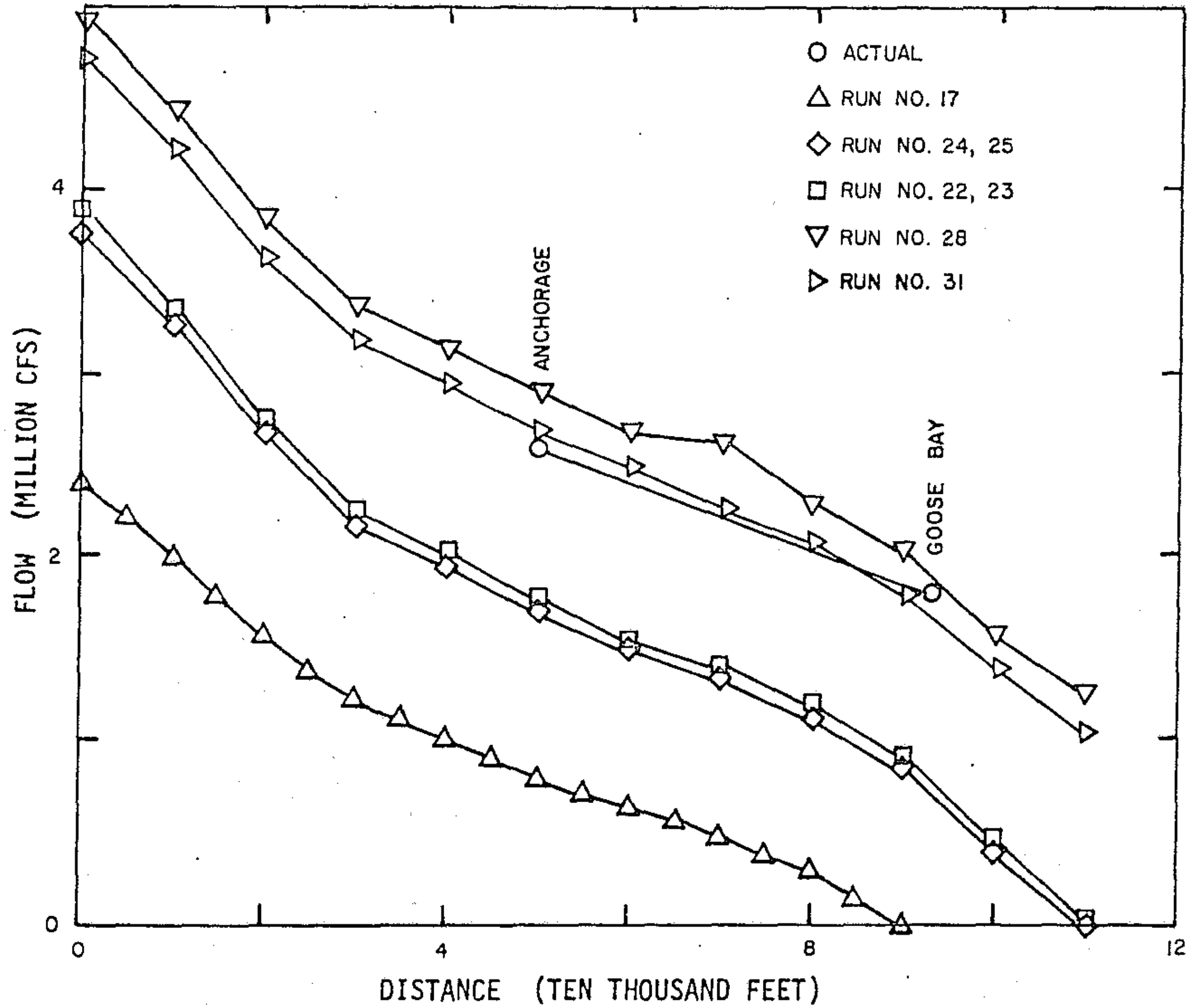


Figure 4 - 7. The maximum flow value as a function of distance for Knik Arm -- field and computed values.

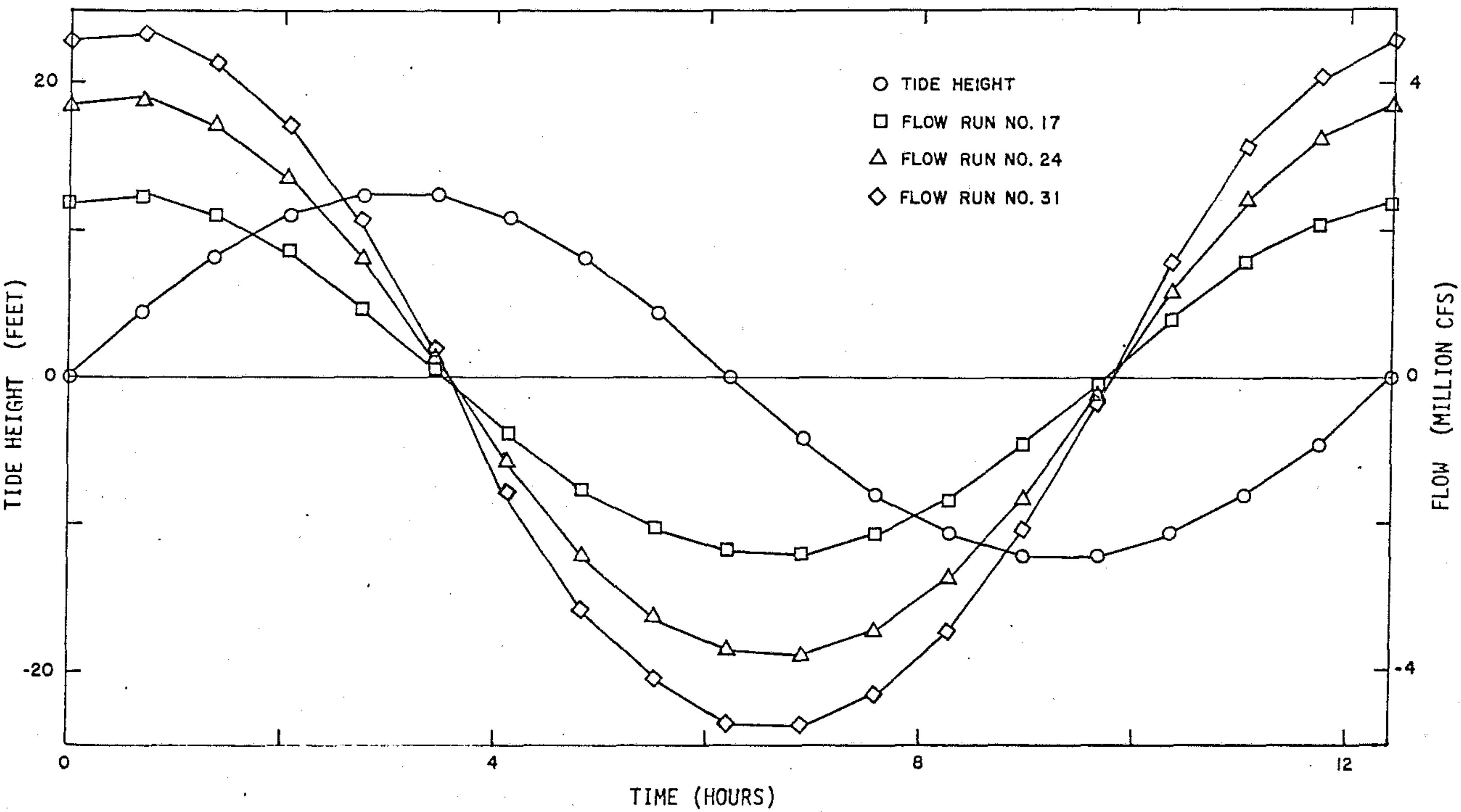


Figure 4-8. Computed tide height and discharge as a function of time for one tidal cycle at Fire Island - Runs 17, 24, and 31.

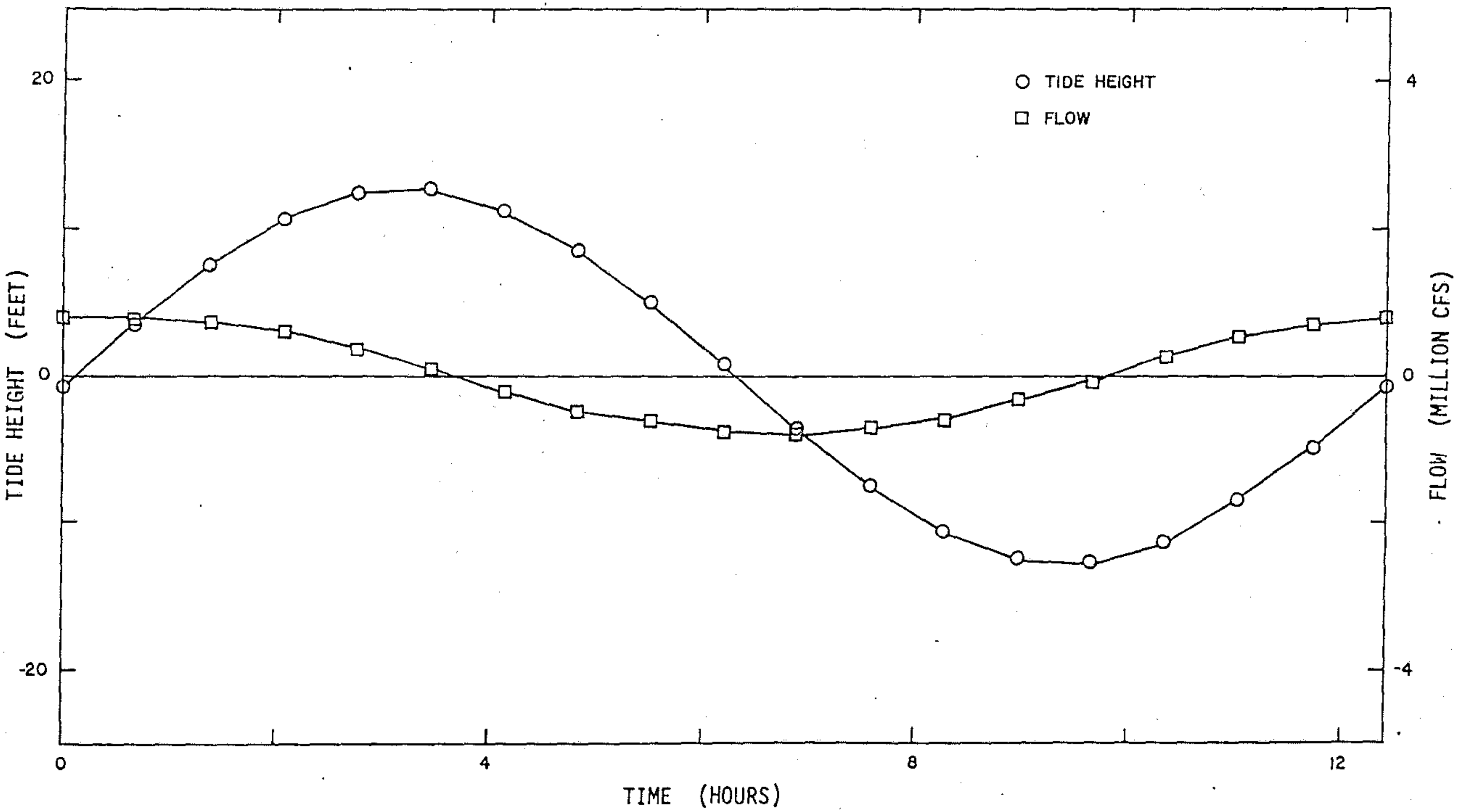


Figure 4 - 9. Computed tide height and discharge as a function of time for one tidal cycle at the Anchorage dock area - Run 17.

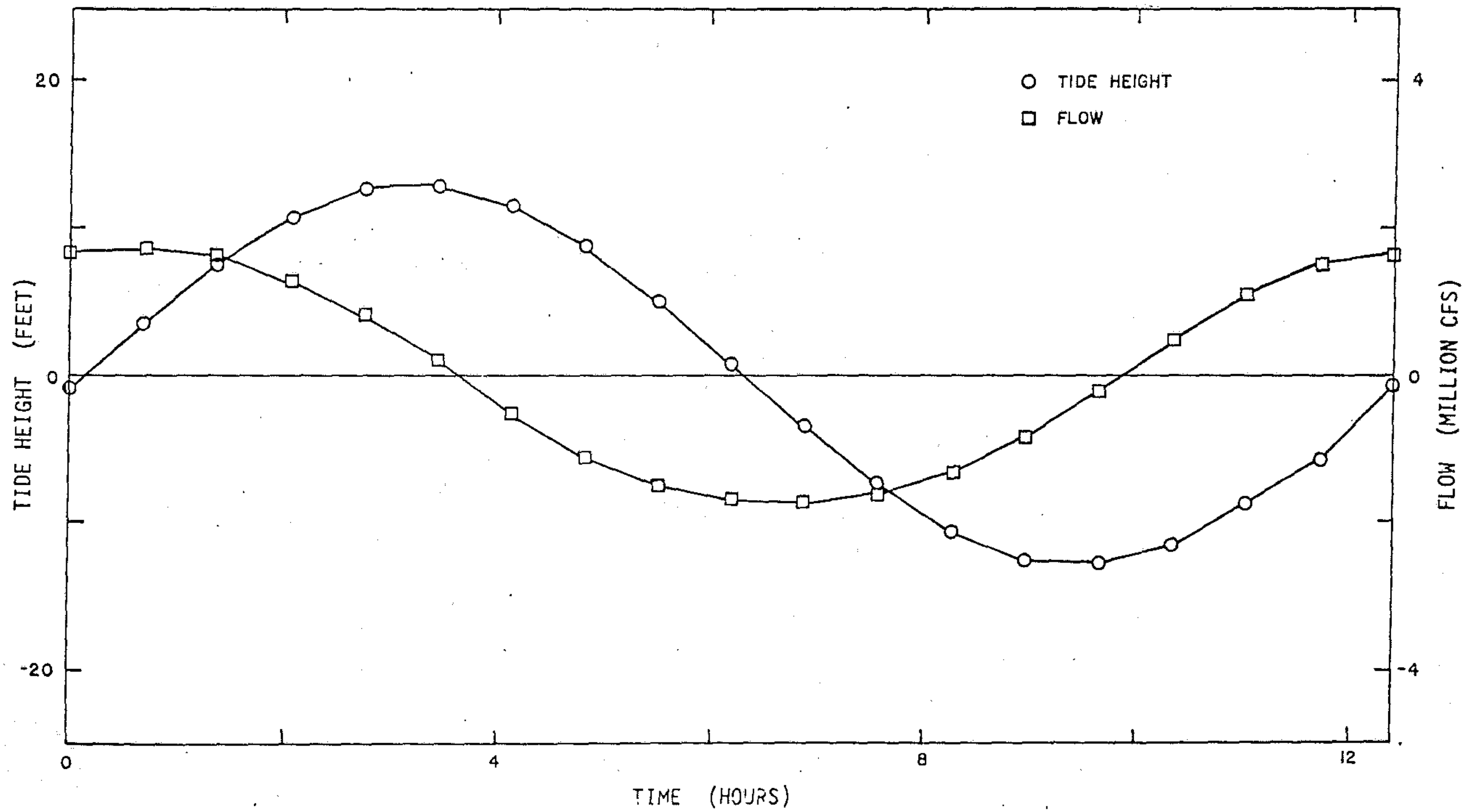


Figure 4 - 10. Computed tide height and discharge as a function of time for one tidal cycle at the Anchorage dock area - Run 24. (Representative of Runs 22 through 25).

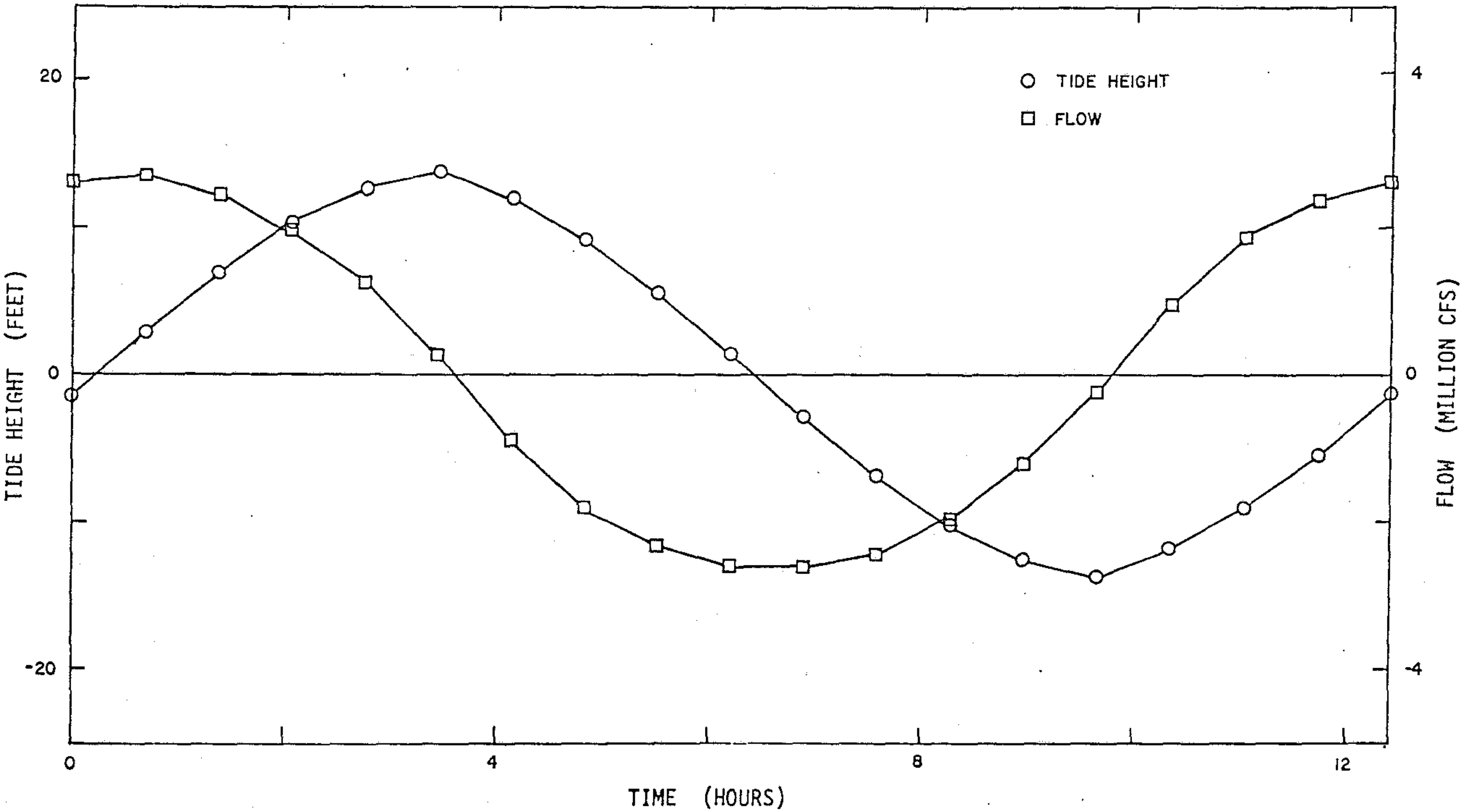


Figure 4 - 11. Computed tide height and discharge as a function of time for one tidal cycle at the Anchorage dock area - Run 31. (Representative of Runs 28 through 31).

It is interesting to note that, although we did not investigate completely the effect of the freshwater flow, the results indicated that the variation of freshwater flow has very little effect on either the tide height or the flow regime in Knik Arm. This is in spite of the fact that Knik Arm receives a flow of freshwater flow variation, mainly from the Knik and Matanuska Rivers, which varies from near zero to 100,000 cubic feet per second. The reason is quite easy to understand - the tide range at Cook Inlet is at such a màgnitude that it causes an immense amount of water to be moving about. The dynamic and initial forces created are of such magnitude that it is simply not affected a great deal by even flow as large as 100,000 cubic feet per second. Of course, this is not to say that the effect of the flow is not felt, because if one were considering a longer term advective flow, this would move down the Inlet at a computed rate of about one-tenth of a foot per second, which again is small compared to the tidal velocities, but is appreciable when considered over a long period of time.

CHAPTER 5 - Summary of Results

The previous pages describe a project which attempted to gain a better understanding of the nature of the hydraulic regime of Cook Inlet and of Knik Arm, Alaska. The primary consideration was to pay special attention to the hydraulic variables which are important to obtain a better understanding of the movement of pollutants in these waters. Thus, we were more interested in the distribution of flows within the Inlet, rather than the other co-variable, the tidal height.

Two basic methods were used to explain the several features of the observed hydraulic regime of two bodies of water -- the simple analytic models, and a rather simple, one-dimensional numerical computer model. The advantages and disadvantages of each model were pointed out in several places in the two chapters devoted to the discussion of these models.

We tend to favor the numerical method, primarily because of its flexibility and ease of application of difficult boundary conditions. Although, as we also pointed out, the analytic methods have much to recommend them, and would be preferable to the numerical methods in several instances.

Several conclusions have been pointed out several places in the text. Here we wish to emphasize the more important areas.

The hydraulic regime may be adequately represented as being one-dimensional, both in Cook Inlet and in Knik Arm. That is, we feel that all important variations in velocity and tidal height will take place primarily in the longitudinal direction of outflow, rather than in the traverse direction of flow to the axis of the inlet and the arm. Although two-dimensional effects certainly exist, the amount of available data does not warrant a very complicated study of them at this time. Furthermore, the one-dimensional effect of the flow variables, for example, velocity, so overwhelms the two-dimensional effects that a one-dimensional consideration would be most useful for most engineering efforts. Two-dimensional effects are certainly important for more subtle changes over longer periods of time, and we do not mean to imply here that any two-dimensional studies are not warranted, but, for some period of time to come, especially in face of the lack of data, most problems can be adequately handled in a one-dimensional frame of reference.

The tidal range although extremely large (approximately up to 30 feet at Anchorage) is not particularly unusual. It is most simply explained as being the result of high tides active at the mouth of the Inlet (15 feet at Seldovia). The resulting amplification is only a factor of 2, a factor which is found in many inlets in the United States, for example, Long Island Sound or the Bay of Fundy. We mention this because many others have attempted to attribute the high tides of the inlet to a variety of factors, the most common being some type of closeness of the inlet length to that of a quarter wave length.

The lack of any adequate data base prevents a great deal of confidence being placed in any type of model study. We think we have adequately pointed this out in the several chapters, especially those relating to the numerical modeling effort of Knik Arm. At the same time this lack of data demands a great deal of flexibility in any type study when applied to the hydraulic regime of the Inlet. For this reason, we feel that the one-dimensional numerical models have much to offer.

The one-dimensional numerical model which we used is well suited for a wide variety of applications. This has been illustrated in several ways. For example, the flexibility in establishing various boundary conditions, either in the flow variables, or in the actual geometric boundaries; especially important is the ample choice one has between specifying any pair of the four flow variables at the end of the region. Thus, if one has only the water surface elevation records, the flows can be computed. Or, if one has elevation records at one point, and flow conditions on another, elevations and flows can be computed. We feel that this is the main advantage of the numerical models over analytic models. Analytic models are difficult to extend to unknown or unobserved flow conditions, such as a narrowing of the channel. Nevertheless, analytic models are quite useful in certain situations, and their applicability to the hydraulic regime of Cook Inlet should be studied further.

We would suggest that further use of our work could be done in the following areas: 1) further development of analytic models of flow for special cases, 2) application of a one-dimensional flow model, 3) experiments with various boundary conditions, such as extreme tides, extreme freshwater floods at the head of the inlet, additions of causeways, the modification of the channel for the reasons of one sort or another, and 4) in conjunction with studies of the type in number 2, a study of effects of the various natural conditions on engineering works in various points in the channel should be carried out. For example, special attention should be given to the effect on existing engineering works, such as platforms, pipelines and wharves.

The water environment of Cook Inlet and its adjacent arms will play an important part in most future economic development of the state, regardless of its location, simply because that under, within, at the surface, and around the shores of the inlet, lie an important part of Alaska's resources economy. It is the site of Alaska's largest city, Anchorage, its only producing oil fields, an important fishery, the largest industrial complex in the state, and most important, Alaska's busiest transportation linkage. As development in the Cook Inlet Basin itself, and Alaska as a whole, increases, there will be an increasing need for more information on the characteristics of the water body. Much of the needed information will be centered around the hydraulic or water flow characteristics. It is particularly important to realize that the Inlet is vital to the whole of Alaska, and that it is unique in so many respects that the standard engineering design criteria cannot simply be transferred intact. Instead, the situation demands a continuing series of research studies and investigations in combination with a high degree of flexibility on the part of engineers.

References

1. Dronkers, J. J., (1964). Tidal Computations in Rivers and Coastal Waters, North-Holland Publishing Co., Amsterdam.

2. Dronkers, J. J., (1969). "Tidal Computations for Rivers, Coastal Areas, and Seas," Proceed. Am. Soc. Civil Engrs., 95:HY1, January.

3. Gaither, W. S., and R. E. Dalton, (1969). "All-Weather Tanker Terminal for Cook Inlet, Alaska," Proceed. Am. Soc. Civil Engrs., 95:WW2, May.

4. Henderson, F. M., (1966). Open Channel Flow, Macmillan, New York.

5. Ippen, A. T., Ed., (1966). Estuary and Coastline Hydrodynamics, McGraw-Hill, New York.

6. Lamb, H., (1945). Hydrodynamics, Dover, New York.

7. Liggett, J. J., and D. A. Woolheiser, (1967). "Difference Solution of the Shallow Water Wave Equation," Proceed. Am. Soc. Civil Engr., 93: EM2, April.

8. Rosenberg, D. H., D. C. Burrell, K. V. Natarajan, and D. W. Hood, (1967). "Oceanography of Cook Inlet," Unpublished Report, Institute of Marine Science, University of Alaska, College, Alaska.

9. United States Coast and Geodetic Survey, (1967). "Tide Tables - West Coast of North and South America, 1969, and Tidal Current Tables - Pacific Coast of North America and Asia, 1969," U. S. Govt. Printing Office, Washington, D. C.

10. Wagner, D. G., C. E. Behlke, and R. S. Murphy, (1969). "A Program for Cook Inlet, Alaska," Report IWR-7, Institute of Water Resources, University of Alaska, College, Alaska.

11. Zetler, B. D., and G. W. Lennon, (1967). "Some Comparative Tests of Tidal Analytical Processes," International Hydrographic Review, 54:1.

Appendix A

Development of the Numeric Computation Routine

The numeric computation of the two flow equations is carried out according to a scheme presented by Dronkers (1969) which he calls the third implicit method. This method was chosen from several given by Dronkers and from many others because of several desirable characteristics. Implicit schemes in general have the favorable characteristics of accuracy and stability (see Ligget and Woolhiser, 1967) and few disadvantages. (Perhaps the only important one is that they require more computation space.) The "third" implicit method has slightly more flexibility in that the distance steps are more flexible than the other methods.

This appendix presents the complete development of the numeric computation scheme from the basic hydrodynamic differential equations to the computational algorithm. The development follows that of Dronkers (1969); but as his is necessarily brief and somewhat incomplete, the full procedure is presented here.

We begin with the basic hydrodynamic equations of motion--conservation of momentum,

$$\frac{\partial h}{\partial x} = \frac{1}{g\ A}\frac{\partial Q}{\partial t} - \frac{|Q|\ Q}{C^2\ A^2\ (a_o + h)} \tag{A-1}$$

and conservation of mass,

$$\frac{\partial Q}{\partial x} = b\ \frac{\partial h}{\partial t}. \tag{A-2}$$

where Q and h are the solution variables,

Q = flow rate past a section, $ft^3\ sec^{-1}$

h = the height variable at a section as defined in Figure A-1, ft.

The other variables are:

g = gravity constant, $ft\ sec^{-2}$,

A = cross-sectional area of a segment, ft^2,

t = time, sec,

C = Chezy's friction coefficient, $ft^{1/2}\ sec^{-1}$,

a_o = mean depth of a segment or cross-section, ft,

x = horizontal distance, ft, and

b = storage or top width of a segment, ft.

The following difference equations are written from Equations A-1 and A-2:

$$h'_{m+1} - h'_m = \frac{-\Delta x_m}{\tau 2 g A_m} \left[(Q'_{m+1} - Q_{m+1}) + (Q'_m - Q_m) \right]$$

$$- \frac{\Delta x_m}{4} \frac{|Q_{m+1} + Q_m| (Q'_{m+1} + Q'_m)}{C^2_m A^2_m a_m} \qquad \text{(A-3)}$$

and

$$Q'_{m+1} - Q'_m = -\frac{\Delta x_m}{2\tau} b_m \left[(h'_{m+1} - h_{m+1}) + (h'_m - h_m) \right]. \qquad \text{(A-4)}$$

The prime notation indicates values at the forward time point, one time step in advance of the present time. The purpose of the solution is to find $h'_1 Q'_1$, $h'_2 Q'_2$, . . . , $h_m Qm$ at successively advancing time points.

The implicit method finds these values as a common solution where each value is found as one of M unknowns in M equations.

In order to condense things somewhat, Equations A-3 and A-4 are rearranged and rewritten in terms of h'_{m+1}, Q'_{m+1}, h'_m, and Q'_m,

$$h'_{m+1} + I_m Q'_{m+1} - h'_m + I_m Q'_m = L_m, \tag{A-5}$$

and

$$\theta_m h'_{m+1} + Q'_{m+1} + {}_m h'_m - Q'_m = J_m; \tag{A-6}$$

thus the "variables of convenience", I_m, θ_m, L_m, and J_m, are defined as:

$$\theta_m = \frac{\Delta x_m b_m}{2\tau},$$

$$I_m = \alpha m + \beta m \, |Q_{m+1} + Q_m| ,$$

$$L_m = \alpha m \, (Q_{m+1} + Q_m) ,$$

$$J_m = \theta m \, (h_{m+1} + h_m), \tag{A-7}$$

$$\alpha_m = \frac{\Delta x_m}{2 \, g A_m},$$

and

$$\beta_m = \frac{\Delta x_m}{4C^2_m A_m a_m}.$$

The solution of the set of equations is found by the "double sweep" method in which unknowns are rewritten in terms of recursive formulas. The first sweep evaluates the parameters of the formulas. The back sweep calculates the desired values of the unknown variables.

The recursion relationships will be developed by successive application to sections 1, 2, and 3.

Writing Equation A-5 for sections 1 and 2 gives

$$I_1 Q_1 = L_1 + h_1 - h_2 - I_1 Q_2$$

or

$$Q_1 = -\frac{1}{I_1} h_2 - Q_2 + \frac{L_1 + h_1}{I_1} \qquad \text{(A-8)}$$

(Since all h_m's and Q_m's are at the advanced time point, we will drop the prime notation temporarily.) Comparing Equation A-8 with the desired recursion relationship gives

$$Q_1 = -q_1 h_2 - t_1 Q_2 + S_1 \qquad \text{(A-9)}$$

$$q_1 = \frac{1}{I_1}$$

$$t_1 = 1 \qquad \text{(A-10)}$$

$$S_1 = \frac{L_1 + h_1}{I_1}$$

and we have defined the first set of recursion parameters. Writing Equation A-6 for sections 1 and 2 gives

$$\theta_1 h_2 + Q_2 + \theta_1 h_1 - Q_1 = J_1 ,$$

or

$$h_2 = \frac{1}{\theta_1} Q_2 - h_1 + \frac{J_1}{\theta_1} + \frac{Q_1}{\theta_1} . \qquad \text{(A-11)}$$

Substituting Equation A-9 into Equation A-11 gives

$$h_2 = -\frac{1}{\theta_1} Q_2 - h_1 + \frac{J_1}{\theta_1} - \frac{q_1}{\theta_1} h_2 - \frac{t_1}{\theta_1} Q_2 + \frac{S_1}{\theta_1} ,$$

or

$$h_2 = -\frac{(t_1 + 1)}{(0_1 + q_1)} Q_2 + \frac{(-\theta_1 h_1 + J_1 + S_1)}{(\theta_1 = q_1)} . \qquad \text{(A-12)}$$

Comparing Equation A-12 to the recursion equation gives (A-13)

$$h_2 = -p_2 Q_2 + r_2 ,$$

$$p_2 = \frac{t_1 + 1}{0_1 + q_1} ,$$

and,

$$r_2 = \frac{(-\theta_1 h_1 + J_1 + S_1)}{(\theta_1 + q_1)} . \qquad \text{(A-14)}$$

The set of four equations, A-9, A-10, A-13, and A-14, form the basis of computing the variables Q_1 and h_2. The relations for points 2 and 3 are found in a similar manner. Thus, we write Equation A-6 for points 2 and 3 and get

$$h_3 = -\frac{1}{\theta_2} Q_3 - h_2 + \frac{1}{\theta_2} Q_2 + \frac{J_2}{\theta_2} . \qquad \text{(A-15)}$$

Rewriting the Equation A-9 for points 2 and 3 results in

$$Q_2 = -q_2 h_3 - t_2 Q_3 + S_2 . \qquad \text{(A-16)}$$

Substituting Equations A-13 and A-16 successively into Equation A-15 gives

$$h_3 \left(1 + p_2 q_2 + \frac{q_2}{\theta_2}\right) = -\left(\frac{1}{\theta_2} + p_2 t_2 + \frac{t_2}{\theta_2}\right) Q_3 + p_2 S_2 - r_2 + \frac{J_2}{\theta_2} + \frac{S_2}{\theta_2} , \qquad \text{(A-17)}$$

or

$$h_3 = \frac{-\left(\frac{1}{\theta_2} + p_2 t_2 + \frac{t_2}{\theta_2}\right)}{\left(1 + p_2 q_2 + \frac{q_2}{\theta_2}\right)} Q_3 + \frac{\left(p_2 S_2 - r_2 + \frac{J_2}{\theta_2} + \frac{S_2}{\theta_2}\right)}{\left(1 + p_2 q_2 + \frac{q_2}{\theta_2}\right)} . \qquad \text{(A-18)}$$

Comparing Equation A-18 to A-13, we thus define

$$p_3 = \frac{(1 + \theta_2 p_2 t_2 + t_2)}{(\theta_2 + p_2 q_2 \theta_2 + q_2)} ,$$

$$\text{(A-19)}$$

$$r_3 = \frac{(\theta_2 p_2 S_2 - r_2 \theta_2 + J_2 + S_2)}{(\theta_2 + \theta_2 p_2 q_2 + q_2)} ,$$

and the next recursion equation,

$$h_3 = -p_3 Q_3 + r_3 , \qquad \text{(A-20)}$$

is formed.

Now, writing Equation A-5 for points 3 and 4,

$$h_4 + I_3 Q_4 - h_3 + I_3 Q_3 = L_3 ,$$

and substituting Equation A-20, we get

$$h_4 + I_3 Q_4 + p_3 Q_3 - r_3 + I_3 Q_3 = L_3$$

or

$$Q_3 \, (p_3 + I_3) = -h_4 - I_3 Q_4 + L_3 + r_3$$

or again

$$Q_3 = - \frac{I}{(p_3 + I_3)} h_4 - \frac{I_3}{(p_3 + I_3)} Q_4 + \frac{(L_3 + r_3)}{(p_3 + I_3)} . \qquad \text{(A-22)}$$

Comparing Equation A-22 to

$$Q_3 = -q_3 h_4 - t_3 Q_4 + S_3 ,$$

we thus define

$$q_4 = \frac{1}{I_4 + p_4} ,$$

$$t_4 = \frac{I_4}{I_4 + p_4} , \qquad \text{(A-23)}$$

$$S_4 = \frac{L_4 + r_4}{I_4 + p_4} .$$

A similar procedure could be followed for points 4 and 5, 5 and 6, . . ., M-1 and M, and the following general expressions for the parameters are deduced:

$$p_m = \frac{(1 + \theta_{m-1} p_{m-1} t_{m-1} + t_{m-1})}{(\theta_{m-1} + \theta_{m-1} p_{m-1} q_{m-1} + q_{m-1})} ,$$

and

$$r_m = \frac{(\theta_{m-1} p_{m-1} S_{m-1} - \theta_{m-1} r_{m-1} + J_{m-1} + S_{m-1})}{(\theta_{m-1} + \theta_{m-1} p_{m-1} q_{m-1} + q_{m-1})} ,$$

for m = 2, 3, . . . , M, and

$$q_m = \frac{1}{I_m + p_m} ,$$

$$t_m = \frac{I_m}{I_m = p_m} , \qquad \text{(A-25)}$$

$$S_m = \frac{L_m + r_m}{I_m + p_m} ,$$

for $m = 1, 2, 3, \ldots, M-1$. At $m = 1$, p_m and r_m are defined as

$$p_1 = 0 \tag{A-26}$$

and

$$r_1 = h_1 . \tag{A-27}$$

The computation of flow and water surface elevation with time and distance proceeds in the following fashion.

Beginning at a point in time, the initial conditions are given for the problem. These are values of h_m and Q_m for $m = 1, 2, \ldots, M$.

In addition, the boundary conditions, h_1 and Q_m or h_m, are given for all points in time.

The effect of the initial conditions dies out, and the steady state solution predominates as the time from the initial time increases. The boundary conditions are the forcing functions or the driving force which determines the nature of the steady state flow.

The first step is computing the values of J_m, L_m, θ_m, and I_m according to the Equation A-7.

Next, the coefficients, q_m, t_m, S_m, p_m, and r_m, in

$$Q'_{m-1} = -q_{m-1} h_m - t_{m-1} Q_m + S_{m-1} \tag{A-28}$$

and

$$h'_m = - p_m Q_m + r_m \tag{A-29}$$

are computed according to Equations A-24 and A-25 for $m = 1, 2, \ldots, M-1$. At $m = M$, just r_m and p_m are computed; and the forward sweep is completed.

The backward sweep begins with the computation of h'_M:

$$h'_M = -p_M Q'_M + r_M ; \tag{A-30}$$

then,

$$Q'_{M-1} = -q_{M-1} h'_M - t_{M-1} Q'_M + S_{M-1} , \tag{A-31}$$

and

$$h'_{M-1} = -p_{M-1} A'_{M-1} + r_{M-1} , \tag{A-32}$$

and so on for $m = M - 1, M - 2, \ldots, 2$.

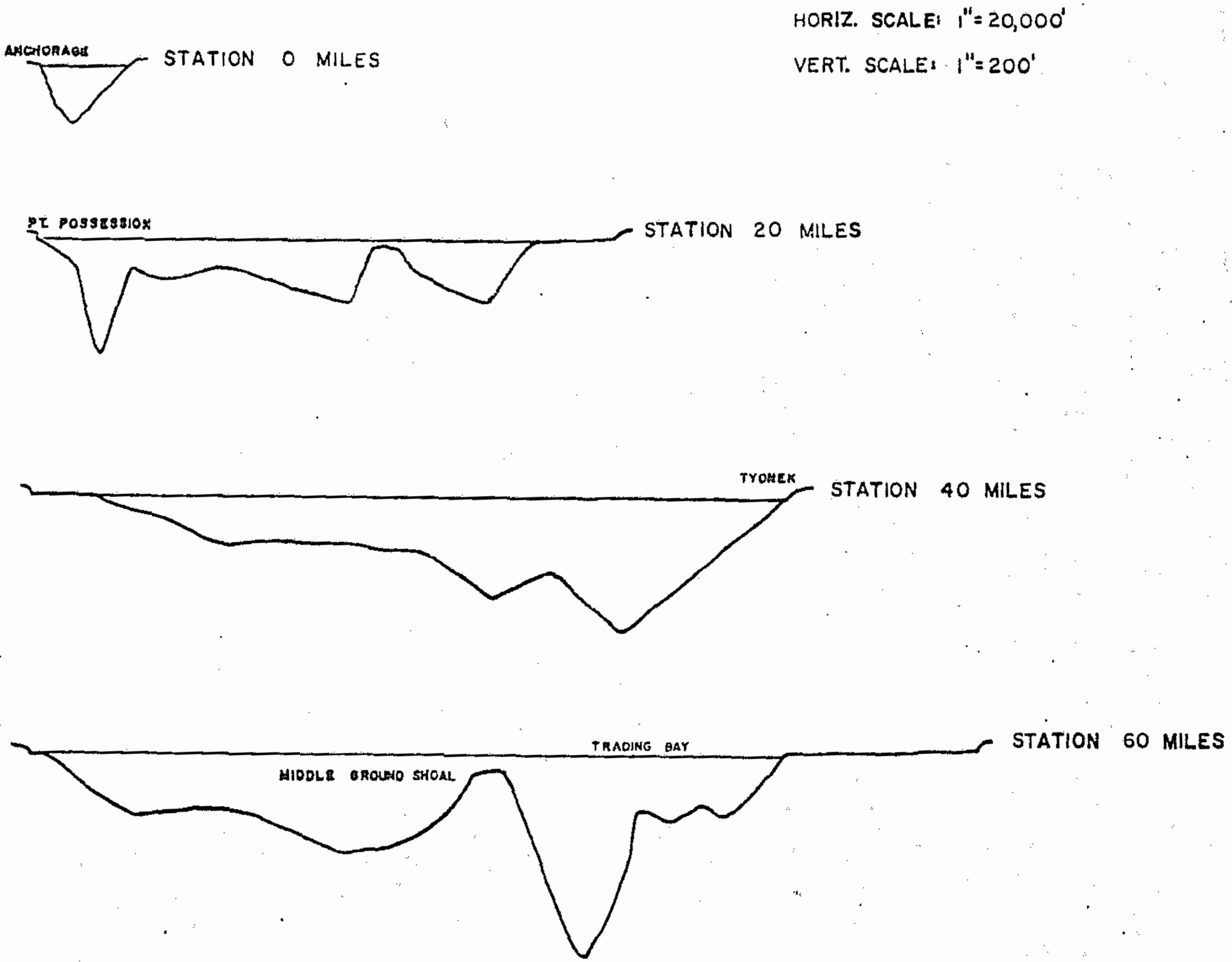
The computation of Q'_1,

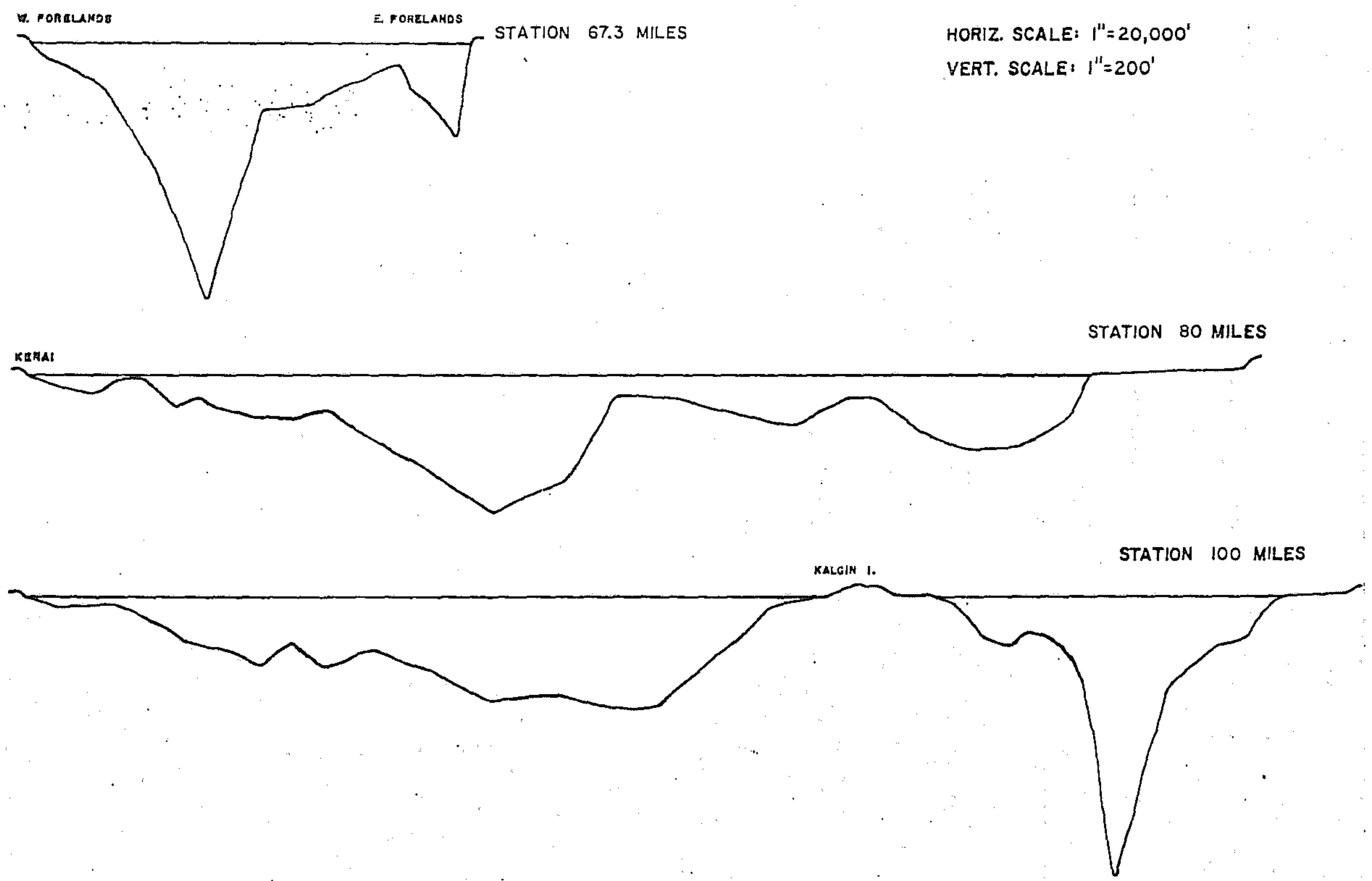
$$Q'_1 = -q_1 h'_2 - t_1 Q'_2 + S_1 , \tag{A-33}$$

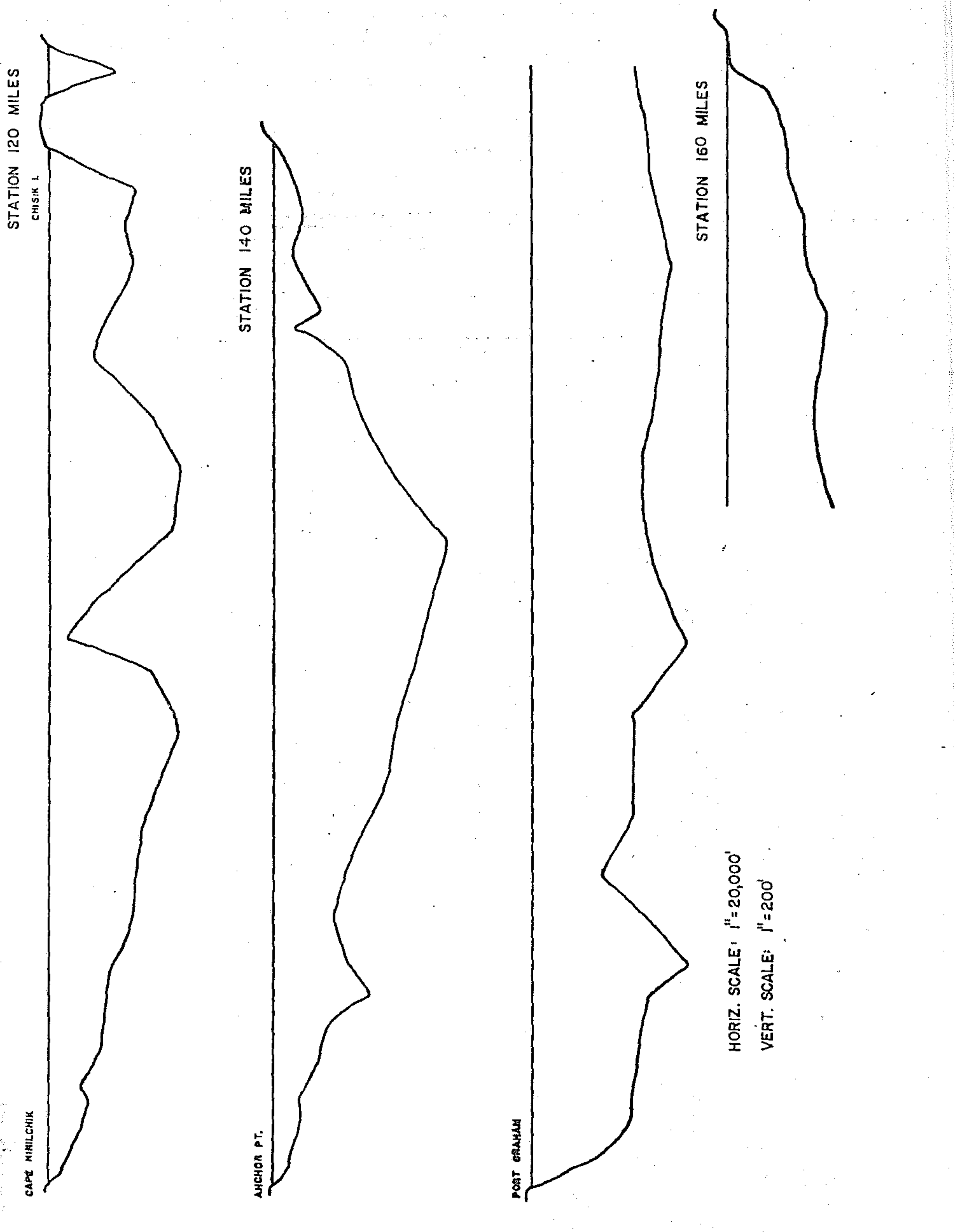
completes the determination of new values. The new values are changed to old values, and the computation repeats itself for the next time step.

Appendix B

Plot of cross section geometry for various stations along Cook Inlet and Knik Arm.


HORIZ. SCALE: 1" = 20,000'
VERT. SCALE: 1" = 200'
ANCHORAGE
STATION 0 MILES
PT. POSSESSION
STATION 20 MILES
TYONEK
STATION 40 MILES
TRADING BAY
MIDDLE GROUND SHOAL
STATION 60 MILES


W. FORELANDS
E. FORELANDS
STATION 67.3 MILES
HORIZ. SCALE: 1" = 20,000'
VERT. SCALE: 1" = 200'
KENAI
STATION 80 MILES
KALGIN I.
STATION 100 MILES


STATION 120 MILES
CHISIK I.
CAPE NINILCHIK
STATION 140 MILES
ANCHOR PT.
STATION 160 MILES
PORT GRAHAM
HORIZ. SCALE: 1"=20,000'
VERT. SCALE: 1"=200'

STATION 67.3 MILES

HORIZ. SCALE: 1"=10,000'

VERT. SCALE: 1"=10,000'

Appendix C

The Computer Program

Source Fortran Program

Input Data

Sample Output

```
// JOB HYDOR, IWR004,       BRITCH        T=10
// OPTION LINK
  ACTION NOMAP
// EXEC FORTRAN
// FTC NODECK,NOLISTX,BCD
      DIMENSIONDX)20*,A(20),C(20),B(20),AS(20),AL(20),BE(20),TH(20)
      DIMENSION H1(20),H2(20),Q1(20),Q2(20),FI(20),FL(20),FJ(20)
      DIMENSION Q(20),T(20),S(20),R(20),P(20),TIDE(40),QUP(40)
C     M=NO OF SECTIONS, ITAU=TIME PER CYCLE, NPER=NO OF PERIODS
C     NCYC=NO OF DIVISIONS PER CYCLE
      READ (1,2)M,ITAU,NPER,NCYC
    2 FORMAT (6X,4I5)
      WRITE (3,5)M,ITAU,NPER,NCYC
      TAU=ITAU
    5 FORMAT(//,3X,15HNO OF SECTIONS=,I5,3X,5HTIME=,I5,3X,
     114HNO OF PERIODS=,I5,3X,15HNO DIV PER CYC=,I5)
C     READ INITIAL SECTION PARAMETERS
C     DX=SECTION LENGTH, A=AVERAGE SECTION AREA, C=CHEZY C
C     B=TOP STORAGE WIDTH, AS=MEAN DEPTH
      MM=M-1
      WRITE(3,3)
    3 FORMAT(//,2X,4HSECT,7X,6HLENGTH,10X,4HAREA,5X,7HCHEZY C,
     28X,5HWIDTH,8X,5HDEPTH,//)
      DO 19 I=1,MM
      READ (1,4) DX(I),A(I),AS(I),C(I),B(I)
    4 FORMAT (3X,5F12.3)
      WRITE (3,6)I,DX(I),A(I),C(I),B(I),AS(I)
    6 FORMAT(I5,2F15.3,F10.3,F15.3,F12.3)
      AL(I)=DX(I)/TAU/64.4/A(I)
      BE(I)=DX(I)/4./C(I)**2/A(I)**2/AS(I)
      TH(I)=DX(I)*B(I)/2./TAU
19    CONTINUE
C     READ INITIAL CONDITIONS OF H AND Q AT EACH POINT
      DO 20 I=1,M
20    READ(1,4)H1(I),Q1(I)
      WRITE(3,7) (H1(I),Q1(I),I=1,M)
      READ (1,11) (TIDE(I),I=1,NCYC)
11    FORMAT (6X,6F10.3)
      WRITE (3,7) (TIDE(I),I=1,NCYC)
      READ (1,69) (QUP(I),I=1,NCYC)
   69 FORMAT (3X,6F11.1)
      WRITE (3,68) (QUP(I),I=1,NCYC)
   68 FORMAT(//,3X,6F11.1)
      LP=1
 7    FORMAT (//,6X,6F10.3)
 C    BEGIN SOLUTION WITH TIME
      DO 50 J=1,NPER
      TIM=J*TAU
      HOUR=TIM/3600.
      H2(1)=TIDE(LP)
      Q2(M)=QUP(LP)
      IF (LP-NCYC) 23,22,22
 22   LP=0
 23   LP=LP+1
      DO 21 I=1,MM
      FI(I)=AL(I)+BE(I)*ABS(Q1(I)+Q1(I+1))
      FL(I)=AL(I)*(Q1(I)+Q1(I+1))
      FJ(I)=TH(I)*(H1(I)+H1(I+1))
 21   CONTINUE
 C    BEGIN RECUR EP PARAM COMP
```

```
      Q(1)=1./FI(1)
      T(1)=1.
      S(1)=(FL(1)+H2(1))/FI(1)
      R(1)=H2(1)
      P(1)=0.0
      DO 25 I=2,M
      K=I-1
      DEM=(TH(K)+TH(K)*P(K)*Q(K)+Q(K))
      P(I)=(1.+TH(K)*P(K)*T(K)+T(K))/DEM
      R(I)=(TH(K)*P(K)*S(K)-TH(K)*R(K)+FJ(K)+S(K))/DEM
      DEM=FI(I)+P(I)
      Q(I)=1./DEM
      T(I)=FI(I)/DEM
25    S(I)=(FL(I)+R(I))/DEM
C     BACK COMPUTATIONS OFR Q AND H
      H2(M)=-P(M)*Q2(M)+R(M)
      MM1=M-2
      DO 27 I=1,MM1
      K=M-I
      L=K+1
      Q2(K)=-Q(K)*H2(L)-T(K)*Q2(L)+S(K)
27    H2(K)=-P(K)*Q2(K)+R(K)
      Q2(1)=-Q(1)*H2(2)-T(1)*Q2(2)+S(1)
C     PRINT RESULTS
      WRITE (3,9) HOUR
9     FORMAT (//,4X,5HTIME=,F10.2)
      WRITE(3,8) (H2(I),I=1,M)
    8 FORMAT(6X,10F12.3,/,8X,10F12.3)
10    FORMAT (8X,10F12.0,/,9X,10F12.0)
      WRITE (3,10) (Q2(I),I=1,M)
      DO 28 I=1,M
      Q1(I)=Q2(I)
28    H1(I)=H2(I)
50    CONTINUE
      STOP
      END
```

DATA DECK FOR RUN NO. 11

```
/*
// EXEC LNKEDT
// EXEC
          9 2483    40
     105600.   38030000.       158.7        100.      239900.
     105600.   19820000.       131.77       100.      157000.
     105600.   19050000.       117.2        100.      162200.
     105600.   13500000.        87.9        100.      153600.
     105600.     9660000.       69.8        100.      141500.
     105600.     6840000.       79.8        100.       85600.
     105600.     4760000.       51.7        100.       92000.
     105600.     2790000.       50.         100.       55800.
       .0        0.0
       .0        0.0
       .0        0.0
       .0        0.0
       .0        0.0
       .0        0.0
       .0        0.0
       .0        0.0
       .0        0.0
          0.0        2.63       4.95       6.67       7.58       7.58
          6.67       4.95       2.63       0.0       -2.63      -4.95
         -6.67      -7.78      -7.78      -6.67      -4.95      -2.63
/*
/+
```

DATA DECK FOR RUN NO. 13

```
/*
// EXEC LNKEDT
// EXEC
         9 2483    40
   105600.   38030000.        158.7        120.      239900.
   105600.   19820000.        131.77       120.      157000.
   105600.   19050000.        117.2        120.      162200.
   105600.   13500000.         87.9        120.      153800.
   105600.    9860000.         69.8        120.      141500.
   105600.    6840000.         79.8        120.       85600.
   105600.    4760000.         51.7        120.       92000.
   105600.    2790000.         50.         120.       55800.
      .0      0.0
      .0      0.0
      .0      0.0
      .0      0.0
      .0      0.0
      .0      0.0
      .0      0.0
      .0      0.0
      .0      0.0
         0.0       2.63       4.95       6.67       7.58       7.58
         6.67      4.95       2.63       0.0       -2.63      -4.95
        -6.67     -7.78      -7.78      -6.67      -4.95      -2.63
/*
/+
```

DATA DECK FOR RUN NO. 17

```
/*
// EXEC LNKEDT
// EXEC
     19 2483    60
   5000.      680000.        29.6        120.       22967.
   5000.      781670.        32.2        120.       24167.
   5000.      690000.        21.1        120.       25467.
   5000.      596670.        23.9        120.       25000.
   5000.      621670.        26.6        120.       23367.
   5000.      611670.        35.6        120.       17167.
   5000.      500000.        40.5        120.       12333.
   5000.      600000.        48.9        120.       12267.
   5000.      558330.        46.5        120.       12000.
   5000.      461670.        37.0        120.       12500.
   5000.      598330.        58.3        120.       10267.
   5000.      703330.        87.6        120.        8033.
   5000.      461670.        60.2        120.        7667.
   5000.      348330.        35.1        120.        9933.
   5000.      315000.        28.6        120.       11000.
   5000.      341670.        28.2        120.       12133.
   5000.      310000.        21.4        120.       14500.
   5000.      361670.        20.1        120.       18000.
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
   .0         0.0
      0.00       4.26       8.01      10.79      12.27      12.27
    -10.79       8.01       4.26       0.00      -4.26      -8.01
     10.79     -12.27     -12.27     -10.79      -8.01      -4.26
/*
/+
```

DATA DECK FOR RUN NO. 22

```
/*
// EXEC LNKEDT
// EXEC
        12 2483    40    18
    10000.     680000.       29.6        120.      31267.
    10000.     690000.       21.1        120.      36667.
    10000.     621670.       26.6        120.      29167.
    10000.     500000.       40.5        120.      13667.
    10000.     558330.       46.5        120.      15000.
    10000.     598330.       58.3        120.      12333.
    10000.     461670.       60.2        120.      10000.
    10000.     315000.       28.6        120.      13667.
    10000.     310000.       21.4        120.      15567.
    10000.     274560.       16.7        120.      28564.
    10000.     126720.       16.4        120.      27984.
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
        0.00       4.26       8.01      10.79      12.27      12.27
       10.79       8.01       4.26       0.00      -4.26      -8.01
       10.79     -12.27     -12.27     -10.79      -8.01      -4.26
/*
/+
```

DATA DECK FOR RUN NO. 23

```
/*
// EXEC LNKEDT
// EXEC
     12 2483    40    18
    10000.     680000.       29.6       120.     31267.
    10000.     690000.       21.1       120.     36667.
    10000.     621670.       26.6       120.     29167.
    10000.     500000.       40.5       120.     13667.
    10000.     558330.       46.5       120.     15000.
    10000.     598330.       58.3       120.     12333.
    10000.     461670.       60.2       120.     10000.
    10000.     315000.       28.6       120.     13667.
    10000.     310000.       21.4       120.     15567.
    10000.     274560.       16.7       120.     28564.
    10000.     126720.       16.4       120.     27984.
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
       0.00       4.26       8.01      10.79      12.27      12.27
      10.79       8.01       4.26       0.00      -4.26      -8.01
      10.79     -12.27     -12.27     -10.79      -8.01      -4.26
/*
/+
```

DATA DECK FOR RUN NO. 24

```
/*
// EXEC LNKEDT
// EXEC
      12  2483    40    18
   10000.     980504.        41.6        120.       29192.
   10000.    1029204.        33.0        120.       33867.
   10000.     919474.        38.6        120.       27717.
   10000.     651992.        52.5        120.       13333.
   10000.     711330.        58.5        120.       14250.
   10000.     727726.        70.3        120.       11814.
   10000.     560670.        72.2        120.        9417.
   10000.     455000.        40.6        120.       13000.
   10000.     487192.        33.4        120.       15300.
   10000.     507852.        28.7        120.       25523.
   10000.     279720.        28.4        120.       22934.
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
     .0         0.0
        0.00       4.26       8.01      10.79      12.27      12.27
       10.79       8.01       4.26       0.00      -4.26      -8.01
       10.79     -12.27     -12.27     -10.79      -8.01      -4.26
/*
/+
```

DATA DECK FOR RUN NO. 25

```
/*
// EXEC LNKEDT
// EXEC
        12 2483    40    18
      10000.       980504.          41.6          120.        29192.
      10000.      1029204.          33.0          120.        33867.
      10000.       919474.          38.6          120.        27717.
      10000.       651992.          52.5          120.        13333.
      10000.       711330.          58.5          120.        14250.
      10000.       727726.          70.3          120.        11814.
      10000.       560670.          72.2          120.         9417.
      10000.       455000.          40.6          120.        13000.
      10000.       487192.          33.4          120.        15300.
      10000.       507852.          28.7          120.        25523.
      10000.       279720.          28.4          120.        22934.
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
       .0         0.0
          0.00        4.26        8.01       10.79       12.27       12.27
         10.79        8.01        4.26        0.00       -4.26       -8.01
         10.79      -12.27      -12.27      -10.79       -8.01       -4.26
/*
/+
```

DATA DECK FOR RUN NO. 28

```
/*
// EXEC LNKEDT
// EXEC
         12 2483   40   18
     10000.     980504.        41.6        120.       29192.
     10000.    1029204.        33.0        120.       33867.
     10000.     919474.        38.6        120.       27717.
     10000.     651992.        52.5        120.       13333.
     10000.     711330.        58.5        120.       14250.
     10000.     727726.        70.3        120.       11814.
     10000.     560670.        72.2        120.        9417.
     10000.     455000.        40.6        120.       13000.
     10000.     487192.        33.4        120.       15300.
     10000.     507852.        28.7        120.       25523.
     10000.     279720.        28.4        120.       22934.
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
        0.00        4.26        8.01       10.79       12.27       12.27
       10.79        8.01        4.26        0.00       -4.26       -8.01
       10.79      -12.27      -12.27      -10.79       -8.01       -4.26
    1254000.    1178384.     960564.     627000.     217694.    -217694.
     627000.    -960564.   -1178384.   -1254000.   -1178384.    -960564.
     627000.    -217694.     217694.     627000.     960564.    1178384.
/*
/+
```

DATA DECK FOR RUN NO. 29

```
/*
// EXEC LNKEDT
// EXEC
        12 4966    80     9
   10000.     980504.        41.6        120.       29192.
   10000.    1029204.        33.0        120.       33867.
   10000.     919474.        38.6        120.       27717.
   10000.     651992.        52.5        120.       13333.
   10000.     711330.        58.5        120.       14250.
   10000.     727726.        70.3        120.       11814.
   10000.     560670.        72.2        120.        9417.
   10000.     455000.        40.6        120.       13000.
   10000.     487192.        33.4        120.       15300.
   10000.     507852.        28.7        120.       25523.
   10000.     279720.        28.4        120.       22934.
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
     .0        0.0
         0.00       8.01       12.27      10.79       4.26      -4.26
       -10.79     -12.27       -8.01
   1254000.     960564.     217694.    -627000.   -1178384.  -1178384.
    627000.     217694.     960564.
/*
/+
```

DATA DECK FOR RUN NO. 30

```
/*
// EXEC LNKEDT
// EXEC
        12 1242    80    36
   10000.     980504.          41.6        120.      29192.
   10000.    1029204.          33.0        120.      33867.
   10000.     919474.          38.6        120.      27717.
   10000.     651992.          52.5        120.      13333.
   10000.     711330.          58.5        120.      14250.
   10000.     727726.          70.3        120.      11814.
   10000.     560670.          72.2        120.       9417.
   10000.     455000.          40.6        120.      13000.
   10000.     487192.          33.4        120.      15300.
   10000.     507852.          28.7        120.      25523.
   10000.     279720.          28.4        120.      22934.
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
     .0       0.0
         0.00        2.14        4.26        6.15        8.01        9.42
        10.79       11.56       12.27       12.30       12.27       11.56
        10.79        9.42        8.01        6.15        4.26        2.14
         0.00       -2.14       -4.26       -6.15       -8.01       -9.42
       -10.79      -11.56      -12.27      -12.30      -12.27      -11.56
       -10.79       -9.42       -8.01       -6.15       -4.26       -2.14
   1254000.    1234939.    1178384.    1085989.     960564.     806046.
    627000.     428893.     217694.          0.    -217694.    -428893.
    627000.    -806046.    -960564.   -1085989.   -1178384.   -1234939.
   1254000.   -1234939.   -1178384.   -1085989.    -960564.    -806046.
    627000.    -428893.    -217694.          0.     217694.     428893.
    627000.     806046.     960564.    1085989.    1178384.    1234939.
/*
/+
```

DATA DECK FOR RUN NO. 31

```
/*
// EXEC LNKEDT
// EXEC
       12 2483    40    18
     10000.     980504.        41.6       120.      29192.
     10000.    1029204.        33.0       120.      33867.
     10000.     919474.        38.6       120.      27717.
     10000.     651992.        52.5       120.      13333.
     10000.     711330.        58.5       120.      14250.
     10000.     727726.        70.3       120.      11814.
     10000.     560670.        72.2       120.       9417.
     10000.     455000.        40.6       120.      13000.
     10000.     487192.        33.4       120.      15300.
     10000.     507852.        28.7       120.      25523.
     10000.     279720.        28.4       120.      22934.
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
       .0       0.0
        0.00       4.26       8.01      10.79      12.27      12.27
       10.79       8.01       4.26       0.00      -4.26      -8.01
       10.79     -12.27     -12.27     -10.79      -8.01      -4.26
    1000000.     939690.    766040.    500000.    173650.   -173650.
     500000.    -766040.   -939690.  -1000000.   -939690.   -766040.
     500000.    -173650.    173650.    500000.    766040.    939690.
/*
/+
```

// EXEC

NO OF SECTIONS= 12 TIME= 2483 NO OF PERIODS= 40 NO DIV PER CYC= 18

SECT	LENGTH	AREA	CHEZY C	WIDTH	DEPTH
1	10000.000	980504.000	120.000	29192.000	41.600
2	10000.000	1029204.000	120.000	33867.000	33.000
3	10000.000	919474.000	120.000	27717.000	38.600
4	10000.000	651992.000	120.000	13333.000	52.500
5	10000.000	711330.000	120.000	14250.000	58.500
6	10000.000	727726.000	120.000	11814.000	70.300
7	10000.000	560670.000	120.000	9417.000	72.200
8	10000.000	455000.000	120.000	13000.000	40.600
9	10000.000	487192.000	120.000	15300.000	33.400
10	10000.000	507852.000	120.000	25523.000	28.700
11	10000.000	279720.000	120.000	22934.000	28.400

0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	4.260	8.010	10.790	12.270	12.270
10.790	8.010	4.260	0.0	-4.260	-8.010
-10.790	-12.270	-12.270	-10.790	-8.010	-4.260
1000000.0	939690.0	766040.0	500000.0	173650.0	-173650.0
-500000.0	-766040.0	-939690.0	-1000000.0	-939690.0	-766040.0
-500000.0	-173650.0	173650.0	500000.0	766040.0	939690.0

```
TIME=     0.65
         0.0           -0.068       -0.134       -0.210       -0.321
          -1.241         -1.658
         531881.        535885.      549674.      568886.      583137.
          866093.       1000000.

TIME=     1.38
         4.260          3.853        3.526        3.231        2.874
          1.195          0.834
        3709504.       3228528.     2711434.     2315081.     2136920.
         1167255.        939690.

TIME=     2.07
         8.010          7.689        7.405        7.160        6.863
          5.454          5.176
        4286416.       3840480.     3314373.     2878640.     2666074.
         1163269.        766040.
```

```
          -0.425       -0.531       -0.673       -0.858       -1.042

          604535.      627277.      650115.      690194.      748731.


           2.590        2.350        2.064        1.706        1.412

         1958720.     1818461.     1711929.     1573172.     1418582.


           6.652        6.492        6.287        5.930        5.643

         2435081.     2235902.     2081257.     1860117.     1599620.
```

Appendix D

List of Symbols and Units.

Q	Flow discharge	$ft^3 - sec^{-1}$
h	Water surface elevation	ft
x	distance along the channel	ft
t	time	sec
b	channel width	ft
g	gravity parameter	$ft - sec^{-2}$
A	cross section area	ft^2
C	Chezy's C, a roughness coefficient wave speed of propagation	$ft^{1/2} - sec^{-1}$ $ft - sec^{-1}$
a_o	mean depth of flow	ft
u	velocity	ft
L	wave length	ft
T	wave period	sec
σ	wave frequency number	sec^{-1}
K	wave number	ft^{-1}
1	channel length	ft
a	wave amplitude	ft
ℓ	channel length	ft